

## Lesson

## 8-5A

Calculating with  
Scientific Notation

► **BIG IDEA** Multiplication and division with numbers in scientific notation can be done using the Product of Powers and Quotient of Powers Properties.

Very large or very small positive numbers are often written in scientific notation. Recall that in scientific notation, a number is represented by  $x \times 10^n$ , where  $n$  is an integer and  $1 \leq x < 10$ . Consider the amount of rice consumed in the world. It is a large number.

**Example 1**

Worldwide rice production in 2009 was about 678 million metric tons. (1 metric ton = 1000 kg  $\approx$  2205 lb). The world population in 2009 was about 6.8 billion people. What was the average rice consumption in the world in 2009, in pounds per person?

**Solution** Average consumption in pounds per person is calculated by dividing total rice production (in pounds) by the world population. By writing both numbers in scientific notation you can simplify the calculation.

$$678 \text{ million metric tons} \approx 678 \text{ million metric tons} \cdot \frac{2205 \text{ lb}}{1 \text{ metric ton}} \approx 678 \cdot 1,000,000 \cdot 2205 \text{ lb}$$

Rewrite these numbers in scientific notation.

$$(6.78 \times 10^2) \cdot 10^6 \cdot (2.205 \times 10^3)$$

Use the Commutative Property of Multiplication and the Product of Powers Property to rewrite the number of pounds of rice.

$$(6.78 \cdot 2.205) \times (10^2 \cdot 10^6 \cdot 10^3) \approx 14.95 \times 10^{11}$$

$$6.8 \text{ billion people} = 6.8 \times 10^9 \text{ people}$$

$$\frac{14.95 \times 10^{11}}{6.8 \times 10^9} = \frac{14.95}{6.8} \times \frac{10^{11}}{10^9} \quad \text{Arithmetic}$$

$$\approx 2.2 \times 10^{11-9} \quad \text{Division, Quotient of Powers Property}$$

$$\approx 2.2 \times 10^2 \quad \text{Subtraction}$$

Average rice consumption in 2009 was about 220 pounds per person.

## Scientific Notation without a Calculator

As shown in Example 1, with some numbers, it is possible to use the Product and Quotient of Powers Properties to do some scientific notation calculations in your head.

### Example 2

Compute  $(3.1 \times 10^{-4})(5 \times 10^{-3})$  without using a calculator. Write the answer in scientific notation.

#### Solution

$$\begin{aligned}
 & (3.1 \times 10^{-4})(5 \times 10^{-3}) \\
 &= (3.1 \cdot 5) \times (10^{-4} \cdot 10^{-3}) \quad \text{Commutative \& Associative Properties} \\
 &= 15.5 \times 10^{-7} \quad \text{Multiplication and Product of Powers} \\
 &= 1.55 \times 10^{-6} \quad \text{Rewrite in scientific notation}
 \end{aligned}$$

### Activity

Fill in the table without using a calculator.

Problem	Mental calculation	Answer in scientific notation
$(8 \times 10^9)(4 \times 10^6)$	$32 \times 10^{15}$	?
$(4.5 \times 10^{12})(2 \times 10^{-8})$	?	?
$\frac{3.5 \times 10^{15}}{5 \times 10^6}$	$0.7 \times \underline{\quad ? \quad}$	?
$\frac{3.5 \times 10^{-3}}{2 \times 10^{-10}}$	?	?
$\frac{2.4 \times 10^4}{3.6 \times 10^{-4}}$	?	?

## Scientific Notation with a Calculator

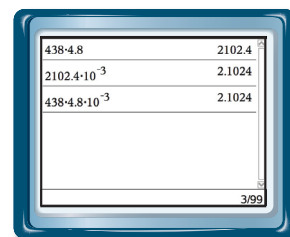
Different calculators have different ways of entering and displaying scientific notation. Some calculators have a special button [EE] to indicate scientific notation, while others require you to write out the entire expression. Take time to determine how your calculator operates.

**Example 3**

A sheet of medium quality paper has thickness  $4.8 \times 10^{-3}$  inch. If this paper is used for a 438-sheet book, how thick will the book be (without the cover)?

**Solution 1** Find  $438 \cdot 4.8$ , then multiply that answer by  $10^{-3}$ .  
 $2102.4 \times 10^{-3} = 2.1024$ .

**Solution 2** Enter  $438 \cdot 4.8 \times 10^{-3}$  in a calculator. The display is 2.1024. The thickness of the book will be about 2.1 inches.



QY

**Questions****COVERING THE IDEAS**

- In 2000, worldwide rice production was about 409 million metric tons. The world population in 2000 was about 6.12 billion.
  - What was the average consumption of rice per person in 2000?
  - Has average rice consumption increased or decreased since 2000?
  - What is the percent increase or decrease of average rice consumption since 2000?

In 2–6, give the answer in scientific notation. Do not use a calculator.

- $(5.1 \times 10^5)(3 \times 10^7)$
- $(2.5 \times 10^{-8})(8 \times 10^{12})$
- $\frac{3.9 \times 10^{16}}{3 \times 10^8}$
- $\frac{1.5 \times 10^{-3}}{7.5 \times 10^{-6}}$
- $\frac{10^{-6}}{4 \times 10^5}$
- Explain how you would compute  $(500)(1.2 \times 10^6)$  without using a calculator.
  - Give the answer in scientific notation.
- Explain two ways to calculate  $(276)(3.2 \times 10^{-8})$  using a calculator.
  - Give the answer in scientific notation.

**QY**

- What does your calculator display for  $25^{25}$ ? Write your answer in scientific notation using two decimal places.
- What does your calculator display for  $25^{-25}$ ? Write your answer in scientific notation using two decimal places.

### APPLYING THE MATHEMATICS

9. A normal heart rate is from 60 to 100 beats per minute. Assume your rate is 75 beats per minute.
  - a. In scientific notation, how many beats is that in a year?
  - b. How long will it take for your heart to beat 1 billion times?
10. One hydrogen atom is about  $5 \times 10^{-8}$  mm in diameter. A human hair is about  $2.5 \times 10^{-3}$  cm in diameter. About how many hydrogen atoms would fit on a cross section of a human hair?
11. In June, 2010, the supercomputer Roadrunner demonstrated the ability to make  $10^{18}$  calculations in one second. This is called a *petaflop*.
  - a. At this rate, for one day, how many calculations will Roadrunner complete?
  - b. You have 26 homework problems. Assuming each problem requires 5 calculations, how long will it take Roadrunner to do your homework?

In 12 and 13, solve for  $x$ . Express your answer in scientific notation.

12.  $x(3.2 \times 10^5) = 8.4 \times 10^{-6}$

13.  $\frac{1.8 \times 10^6}{x} = 4.5 \times 10^3$

14. Write two numbers in scientific notation whose sum you could easily add in your head. Explain what makes the problem a mental math problem.

### QY ANSWER

a.  $8.88 \times 10^{34}$

b.  $1.13 \times 10^{-35}$