Lesson

14-4 When are Figures Congruent?

Vocabulary

congruence transformation

isometry

BIG IDEA Figures are congruent if and only if they are related by a translation, reflection, or rotation, or some combination of these transformations.

Recall from Lesson 14-1 that congruent figures are figures with the same size and shape and that a translation image is always congruent to its preimage. In Lessons 14-2 and 14-3, you saw that both reflection images and rotation images are congruent to their preimages. Because the image of a figure under a translation, reflection, or rotation is congruent to its preimage, translations, reflections, and rotations are examples of *congruence transformations*. A **congruence transformation** is a transformation under which the image and preimage are congruent. A congruence transformation is also called an **isometry**.

All congruence transformations map lines to lines and preserve distance and angle measure. Not all transformations are congruence transformations. In Lesson 14-8, you will learn about another type of transformation that does not preserve distance and so is not a congruence transformation.

If two figures are congruent, then there is a congruence transformation mapping one figure onto the other.

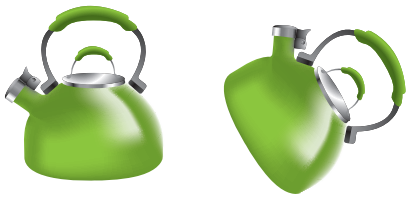
Example

Identify the congruence transformation or combination of congruence transformations that maps the left figure onto the right.

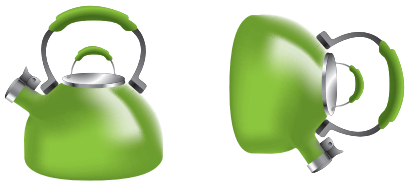
a.

b.

c.

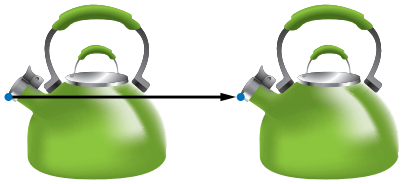


d.

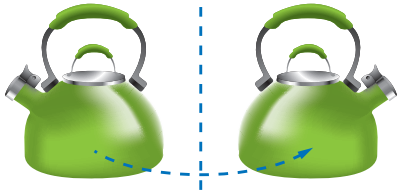


Solution

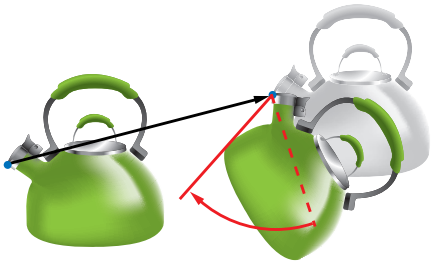
a. The left object can be slid onto the right object, so a translation maps one tea kettle onto the other.



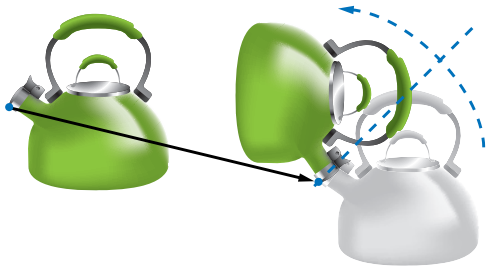
b. The right object is a mirror image of the left object, so a reflection maps one tea kettle onto the other.



c. The left object can be slid so that one point matches up with a point on the right object, then turned to get the whole tea kettle to match. Thus, using a translation and then a rotation maps one tea kettle onto the other.



d. The left object can be slid so that one point matches up with the right object, then reflected to get the whole tea kettle to match. Thus, using a translation and then a reflection maps one tea kettle onto the other.



As in Parts c and d in the Example, it sometimes takes a combination of reflections, rotations, and translations to map one congruent figure onto another. One strategy for finding a congruence transformation from one figure to a congruent figure is to translate the first figure so that at least one point coincides with a corresponding point on the other, then reflect or rotate to get the whole figure to coincide.

Activity

MATERIALS large and small sheets of paper, scissors, and tape

Step 1 Working with a partner, each student should cut out four copies of the same shape from a small sheet of paper.

Step 2 Place the shapes on a large sheet of paper in varying positions and tape them down. Label one shape "*A*".

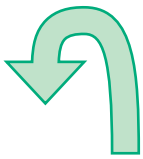
Step 3 Trade large sheets with your partner.

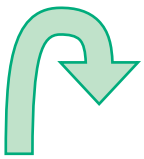
Step 4 Identify congruence transformations that map shape *A* onto each other shape.

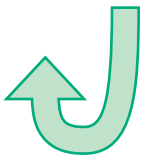
Questions

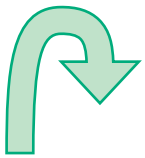
COVERING THE IDEAS

In 1–3, the figure is an image of the original figure at the right. Tell which transformation was applied to the original figure to get the image.

1. 

2. 

3. 



original

4. **Fill in the Blank** Two figures F and G are __?__ if and only if G is the image of F under an isometry.

5. You draw a triangle with sides measuring 3 cm, 4 cm, and 5 cm, with the longest side horizontal. Your friend draws a triangle with sides measuring 3 cm, 4 cm, and 5 cm, with the shortest side horizontal. Are the triangles congruent? Why or why not?

In 6 and 7, suppose you draw a rectangle with dimensions 3 centimeters and 4 centimeters.

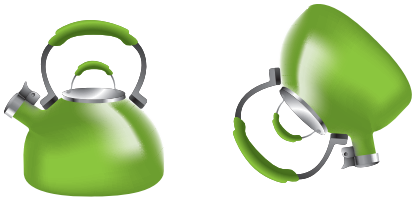
6. A classmate draws a rectangle with dimensions 3 inches and 4 inches. Is your rectangle congruent to this classmate's rectangle?

7. Another classmate draws a rectangle with dimensions 4 centimeters and 3 centimeters. Is this classmate's rectangle congruent to yours?

In 8 and 9, describe a congruence transformation that maps one tea kettle onto the other.

8.

9.



APPLYING THE MATHEMATICS

In 10 and 11, describe a congruence transformation that maps one image onto the other.

10.

11.

In 12 and 13, describe a congruence transformation that maps one image onto the other.

12.

13.

In 14–17, plot the points $A = (1, 3)$, $B = (7, 2)$, and $C = (4, 8)$ and their images, A', B', C' under under the given transformation and determine whether the given transformation is a congruence transformation. Explain your answer.

14. The transformation T with $T(x, y) = (-x, y - 1)$.

15. The transformation T with $T(x, y) = (-y, -x)$.

16. The transformation T with $T(x, y) = (2x, 2y)$.

17. The transformation T with $T(x, y) = (x + 4, 3 - y)$.