Lesson

6-10A Equivalent Expressions

Vocabulary

algebraic expression

evaluating the expression

equivalent expressions

BIG IDEA You can test whether algebraic expressions are equivalent by using substitution, creating tables, or applying properties of operations.

Algebra enables you to approach problems in many different ways. A problem that might be difficult to solve using one approach may be more easily solved using a different approach.

Von was given the problem to calculate the perimeter of a building that was rectangular in shape, $L = 160$ feet long and $W = 40$ feet wide. She drew a picture and added the lengths of the four sides: $L + W + L + W = 160 + 40 + 160 + 40 = 400$ feet.

Charlie looked at the same problem and did it a different way. He said: I add the length and width: $L + W = 160 + 40 = 200$. Then I multiply the sum by 2. So Charlie multiplied $2(L + W) = 2(160 + 40) = 2(200) = 400$ feet, and got the same answer that Von found.

Will Charlie's way always work no matter what numbers are used for the length and width?

Algebraic Expressions

If an expression contains a variable alone or numbers and variables that are combined using the operations of arithmetic, it is called an **algebraic expression**. Some examples of algebraic expressions are $a + 3b$, $2.4 \cdot x - y$, $-m + 2\frac{7}{12}$, and n.

You can find the value of an algebraic expression by substituting the same number for every instance of each variable in the expression. This is called **evaluating the expression**. If an algebraic expression contains more than one variable, you should pick a number for each variable.

For example, if you evaluate the expression $a + 3b$ when a equals 11 and b equals -2, the value of the expression is 5. When a equals 29 and b equals 1, the value of the expression is 32.

Suppose you have two algebraic expressions which contain the same variables. You can evaluate these expressions using the same number for the same variable in both. If the two expressions result in the same value for every number that is used, then they are called **equivalent expressions**.

Example 1

Test whether $L + W + L + W$ and $2(L + W)$ are equivalent.

Solution

Method 1: Use any numbers to test for equivalence. Let $L = 7$ and $W = 6.5$. Evaluate each expression.

When $L = 7$ and $W = 6.5$,
then $L + W + L + W = 7 + 6.5 + 7 + 6.5 = 27$.

When $L = 7$ and $W = 6.5$,
then $2(L + W) = 2(7 + 6.5) = 2 \bullet 13.5 = 27$.

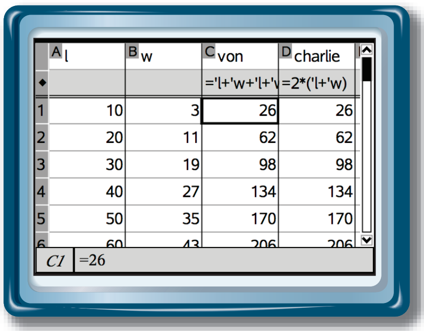
You can repeat this process and test other numbers.
Technology can allow you to test many numbers at once, as seen in Method 2.

Method 2: Use a speadsheet to quickly make a table.
Enter L into the first column, W into the second column.

For Von's expression enter into the third column $L + W + L + W$.

For Charlie's expression enter into the fourth column $2(L + W)$.

We set a table starting at 10 for L with increments of 10 and starting at 3 for W with increments of 8.



	A l	B w	C von	D charlie
◆			='l+'w+'l+'v	=2*('l+'w)
1	10	3	26	26
2	20	11	62	62
3	30	19	98	98
4	40	27	134	134
5	50	35	170	170
6	60	43	206	206

C1 =26

The numbers located in the first two columns are each substituted into the two expressions. The results are automatically calculated and displayed in the third and fourth columns. In each case, the values of the expressions are equal.

Methods 1 and 2 show that the two expressions seem to be equivalent. But, how can you be sure the expressions always have the same value? The answer is to use the properties of operations that are true for all real numbers.

Method 3: Use properties to test equivalence.

$2(L + W) = 2L + 2W$	Use the Distributive Property.
$= L + L + W + W$	Interpret multiplication as repeated addition.
$= L + W + L + W$	Use the Commutative Property of Addition.

By properties of operations, they are equivalent expressions.

Properties of operations are powerful because they can show that a pattern is true for all real numbers. But the other methods are also useful too. Testing specific numbers, either by hand or in a table, can help you decide if two expressions *seem* equivalent. These methods can often help you detect a counterexample. Testing numbers is also a good way to catch your own mistakes.

Example 2

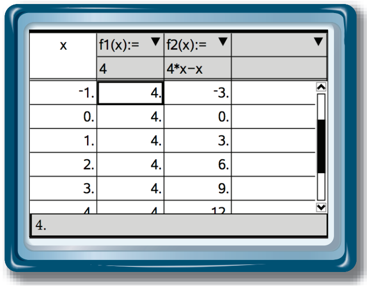
A common error that some students make is to think that $4x - x$ is equivalent to 4 for all values of x. Here are two ways to show that these expressions are not equivalent.

Solution

Method 1: Substitute a value for x to show that $4x - x$ is not equal to 4.

Let $x = 0$. $4x - x = 0$ and $4 = 4$ but $0 \neq 4$.

Method 2: Use a graphing calculator to create a table of values for $Y1 = 4x - x$ and $Y2 = 4$.



x	f1(x):= 4	f2(x):= 4*x−x
-1.	4.	-3.
0.	4.	0.
1.	4.	3.
2.	4.	6.
3.	4.	9.
4.	4.	12.

4.

You should find that for almost all values of x, the expressions do not have the same values. Therefore, $4x - x$ is not equivalent to 4. You could also simplify $4x - x$ to $3x$, which clearly is not 4 for every value of x.

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Example 3

Are $-x^2$ and $(-x)^2$ equivalent expressions? If so, explain. If not, provide a counterexample.

Solution Pick a value for x. Suppose you pick 7. Then $-x^2 = -7^2$ and $(-x)^2 = (-7)^2$.

For -7^2, follow the rules for order of operations and square 7 *before* taking the opposite.

If $x = 7$, then $-x^2 = \underline{\ ?\ }$	Substitute.
$= -(\underline{\ ?\ })(\underline{\ ?\ })$	Evaluate powers first.
$= \underline{\ ?\ }$	Simplify.

For $(-7)^2$, follow the order of operations and square -7.

If $x = 7$, then $(-x)^2 = \underline{\ ?\ }$	Substitute.
$= (\underline{\ ?\ })(\underline{\ ?\ })$	Evaluate powers first.
$= \underline{\ ?\ }$	Simplify.

Because $\underline{\ ?\ }$ and $\underline{\ ?\ }$ are not equal, the expressions are not equivalent. Therefore, 7 is a counterexample.

Example 3 shows that just one counterexample is enough to show that $-x^2$ and $(-x)^2$ are not equivalent. On the other hand, one or two examples are *not* enough to conclude that two expressions are always equivalent. Rather, when you obtain the same value for each expression after testing several specific numbers, you can only conclude that the expressions *seem* equivalent.

Questions

COVERING THE IDEAS

1. When are two algebraic expressions equivalent?

In 2–4, test the two given expressions for equivalence by substitution or by using a table.

2. $4n - 15$ and $4(n - 4) - 1$
3. $3x^2 + 6x(x + 2)$ and $3x^2 + 6x^2 + 2$
4. $(5 + x)^2$ and $25 + 10x + x^2$
5. Is $3x^2$ equivalent to $(3x)^2$? If so, explain. If not, provide a counterexample.
6. Consider the expressions $x \cdot x$ and $2x$.
   a. Copy and complete the table of values at the right.
   b. Give two values of x for which $x \cdot x = 2x$.
   c. Give two values of x for which $x \cdot x$ does not equal $2x$.

x	$x \cdot x$	$2x$
–3	?	?
–2	?	?
–1	?	?
0	?	?
1	?	?
2	?	?
3	?	?

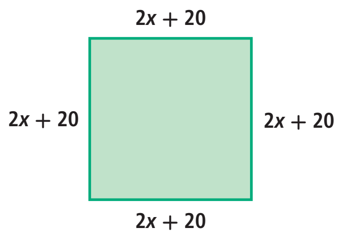
7. An equilateral triangle is a triangle with all three sides of equal length. In finding the perimeter of an equilateral triangle, Marni used this formula: $P = y + y + y$, and Joey used this formula: $P = 3 \cdot y$. Test three sets of numbers in each expression. Do you think the two expressions are equivalent?
8. Copy and complete the table below. Test values of n to find the expression that is not equivalent to the other two. In the last row of the table, pick your own number to test.

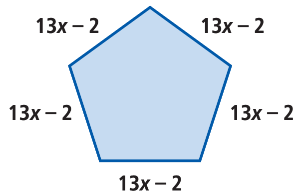
Values of n to test	$2 \cdot n - 4$	$n - 3 + n - 1$	$4 - 2 \cdot n$
0			
40			
–3.5			

9. In determining whether or not the expression $(n + 2)^2$ is equivalent to $n^2 + 2^2$, Jackie tested the number 0 and found that each expression yields a result of 4. Should Jackie conclude that the two expressions are equivalent? Explain.

APPLYING THE MATHEMATICS

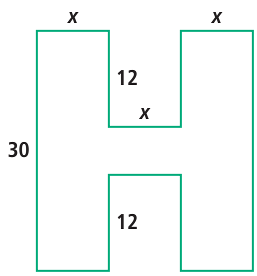
10. The perimeter of the square below equals $2x + 20 + 2x + 20 + 2x + 20 + 2x + 20$. Malek thought the perimeter is $8x + 20$. Is Malek right? Why or why not?





11. Write two equivalent expressions for the perimeter of the regular pentagon at the right.

12. Manuel found the area of the "H" shape below to be $30(3x) - 2(12x)$, while Lina got $30x + 6x + 30x$. Are the two expressions equivalent?
    a. Use a table to find your answer.
    b. Use properties of operations to answer the question.
    c. Let $x = 5$. Find the area of the shape.



In 13 and 14, give a counterexample to show that the two expressions are *not* equivalent.

13. $6 + m$ and $2m - 3(m - 2)$ **14.** $\frac{y}{2} + \frac{3}{2}$ and $\frac{3y}{4}$