

Lesson

8-5A

Inequalities

Vocabulary

double inequality

► **BIG IDEA** Because inequalities may have many solutions, a graph is often used to show them all.

Mathematics teacher Mr. Taylor asked each student in his class to name a number greater than 1,000,000. This was not hard because there are an unlimited number of choices. It is not possible to list them all, but it is possible to represent them with an inequality or a graph.

Example 1

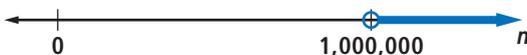
Describe all possible solutions to Mr. Taylor's question.

Solution 1 Use an inequality.

Some possible answers are 1,000,001, 5,362,987, 8,000,000,000, and 1,000,000.1. If n stands for a number greater than 1,000,000 then $n > 1,000,000$ describes all the solutions.

Solution 2 Graph on a number line.

The shaded part of the line represents the solutions. $1,000,000 > 1,000,000$ is not true, so 1,000,000 is not included in the graph. An open circle means the point itself is not part of the graph. A shaded arrowhead indicates the graph continues forever in that direction.

**Greater Than or Equal To**

Suppose Mr. Taylor had allowed numbers greater than *or equal to* 1,000,000. Then 1,000,000 would have been a possible answer. The symbol \geq means, "is greater than or equal to," and a filled-in circle means the point is included in a graph. The inequality describing the solution would then be $n \geq 1,000,000$. Its graph is shown below.



 QY

 QY

What do you think the symbol \leq means?

Double Inequalities

$-8 < -4 \leq -2$ is called a **double inequality** because it has two inequality signs. This is read: “negative eight is less than negative four, which is less than or equal to negative two.” This double inequality contains the inequalities $-8 < -4$ and $-4 \leq -2$, as well as $-8 < -2$.

Be careful: make sure both inequality symbols point in the same direction. For example, do not write $2 < 8 > 1$ because it is not clear whether this means $2 < 1$ or $2 > 1$, and one of these statements is obviously incorrect.

Example 2

On a trip that started at sea level (altitude 0), an airplane reached a maximum altitude of 6.8 miles. Describe all its flying altitudes.

Solution 1 Use an inequality.

The plane started the trip at an altitude of 0 miles and climbed to 6.8 miles, traversing all altitudes between 0 and 6.8 miles. The plane is not flying at 0 miles so 0 is not included in the solution, but it is flying at 6.8 miles so 6.8 is included in the solution. If A stands for altitude in miles, $0 < A \leq 6.8$.

Solution 2 Use a graph.

The endpoints of the graph are 0 and 6.8. A filled-in circle at 6.8 means it is included in the solution, while an open circle at 0 means it is not included.



GUIDED

Example 3

Write an inequality to describe the graph below. Use g as the variable.



Solution The solutions are the numbers from ? to ?.

The solution includes 6 but does not include -2.

One double inequality is ? $< g \leq$?.

Another correct answer is $6 \geq g$? -2.

Questions

COVERING THE IDEAS

1. **True or False** It is possible for an inequality with a variable to have more than one solution.

In 2–5, give three solutions to the inequality.

2. $q > 15$
3. $p < -1.2$
4. $14 \geq r > 13$
5. $-2 < s < 0$

In 6–8, a situation is given.

- a. Write an inequality to describe the situation.
 - b. Give three solutions to the inequality.
 - c. Graph all possible solutions.
6. To ride the Big Thunder Mountain Railroad at Disney World, you must be 40 inches or taller. Use h for height.
 7. To qualify for a reduced-price movie ticket you must be under 12 years of age. Use a for age.
 8. Contestants on *American Idol* must be from 16 to 28 years of age. Use a for age.

In 9–11, write an inequality whose solution is represented by the graph.

9.



10.



11.



12. **Fill in the Blanks** Fill in the inequality symbols that describe the graph below.



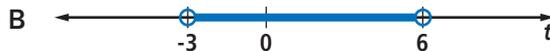
- a. $-8 \underline{\quad ? \quad} y \underline{\quad ? \quad} 4$
- b. $4 \underline{\quad ? \quad} y \underline{\quad ? \quad} -8$

In 13–16, match the sentence with the graph of its solutions.

13. $-3 \leq t < 6$



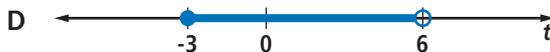
14. $-3 < t < 6$



15. $-3 \leq t \leq 6$



16. $-3 < t \leq 6$



In 17 and 18, write a different inequality that means the same as the given inequality.

17. $h \geq 3\frac{1}{2}$

18. $0.03 < j \leq 0.04$

APPLYING THE MATHEMATICS

19. **Multiple Choice** Which number is not a solution to $-25 \leq v < 4.3$?

- A 4.25
- B $\frac{1}{2}$
- C -40
- D -20

In 20–23, a situation is given.

- a. Write an inequality that represents the situation.
 - b. Graph the inequality on a number line.
20. In 2008, to qualify for the Olympic Trials in the women's 400-meter dash, a runner had to have a time, t , of less than or equal to 51.45 seconds.
 21. Asia has both the highest and lowest elevation on Earth. Mount Everest is 8850 meters high. The Dead Sea has an elevation of -411 meters. Use E to represent possible elevations on Earth.
 22. Water freezes when the temperature, T , is at or below 32°F .
 23. To win a bicycle race, it is necessary to average at least 20 mph. Use s for average speed.

QY ANSWER

The symbol \leq means “is less than or equal to.”