

Lesson

9-9A

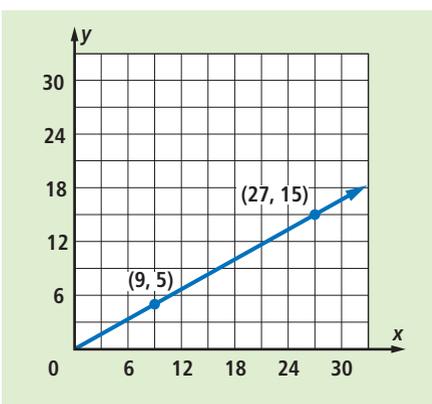
Graphing Proportional Relationships

Vocabulary

unit rate

► **BIG IDEA** The graph of the pairs of positive numbers in a proportional relationship is a ray starting at $(0, 0)$ and passing through $(1, r)$, where $\frac{r}{1}$ is the unit rate.

Consider the pairs of numbers in a true proportion, such as $\frac{9}{5} = \frac{27}{15}$. If the ordered pairs $(9, 5)$ and $(27, 15)$ are graphed, then the line containing the two points will contain the origin $(0, 0)$.



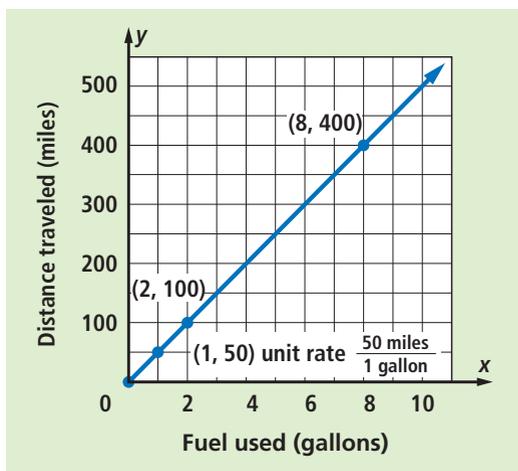
A graph like this one can be found by examining real-world situations involving rates. Activity 1 is about the rate at which a particular car uses gasoline.

Activity 1

- Step 1** Graph the data from the table at the right. Put the fuel used on the horizontal axis and distance traveled on the vertical axis.
- Step 2** Choose three points on the graph. For each point, write the rate $\frac{\text{miles traveled}}{\text{gallons used}}$. Simplify each fraction. Identify the number of miles traveled per gallons used when the denominator is one gallon. This is called the *unit rate*.
- Step 3** Is the relationship between fuel used and distance traveled proportional? Support your answer.

Fuel used (gallons)	Distance traveled (miles)
0	0
0.5	25
2	100
3.5	175
6	300
8	400

Every rate calculated in Step 2 should be 50 miles per gallon, or $50 \frac{\text{miles}}{\text{gallon}}$. This rate can also be written as $\frac{50 \text{ miles}}{1 \text{ gallon}}$. When the denominator of a rate has the number 1, the rate is called a **unit rate**. In the graph of Activity 1, the unit rate is represented by the point (1, 50).



Proportional Relationships and Equations

The graph from Activity 1 has an equation that also describes this proportional relationship. Using y for the number of miles traveled, x for the number of gallons, and the point (8, 400),

$$\frac{y \text{ miles}}{x \text{ gallons}} = \frac{400}{8}.$$

To rewrite this equation in the usual form for graphing, use the Means-Extremes Property.

$$\begin{aligned} \frac{y}{x} &= \frac{400}{8} \\ y \cdot 8 &= x \cdot 400 && \text{Means-Extremes Property} \\ \frac{8 \cdot y}{8} &= \frac{x \cdot 400}{8} && \text{Division Property of Equality} \\ y &= 50x && \text{Simplify} \end{aligned}$$

You can check other points in the table using this equation. For example, using (6, 300), $300 = 50 \cdot 6$. You could also use the equation to find additional values that work in this proportional relationship.

For any pair of points (x, y) and (a, b) on a graph of the equation, $\frac{y}{x} = \frac{b}{a}$ is a true proportion. For example, from the equation above, the points (1.5, 75) and (2.5, 125) are both on the line so $\frac{75 \text{ miles}}{1.5 \text{ gallons}} = \frac{125 \text{ miles}}{2.5 \text{ gallons}}$. They are all equal to the unit rate $\frac{50 \text{ miles}}{1 \text{ gallon}}$.

Caution! Some pairs of values on the graph of a proportional relationship cannot be applied to the proportional situation. Consider the point $(-2, -100)$ on the graph of the line on the previous page. A negative number of gallons used is not possible in this situation. A negative number of miles driven is also not possible, so we must only consider the first quadrant of this graph.

As you have seen above, the table, the graph, the unit rate, and the equation are all ways of representing the same proportional relationship.

STOP QY

QY

If the car has used 10.5 gallons of gasoline, how far has it traveled?

Example

The graph at the right shows a proportional relationship between energy used (in joules) and time (in seconds) of a certain brand of microwave.

- Interpret the point $(20, 30,000)$ on the graph.
- Find the unit rate and explain what it means in terms of the situation.

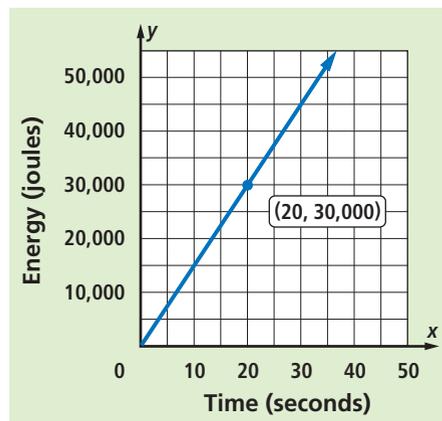
Solution

- The point $(20, 30,000)$ indicates that when the microwave runs for 20 seconds it uses 30,000 joules of energy.
- The unit rate is the rate when the denominator is 1.

Using the point $(20, 30,000)$,

the rate $\frac{\text{energy used}}{\text{seconds}}$ is $\frac{30,000 \text{ joules}}{20 \text{ seconds}}$, which can be simplified to $\frac{1500 \text{ joules}}{1 \text{ second}} = 1500 \text{ joules per second}$.

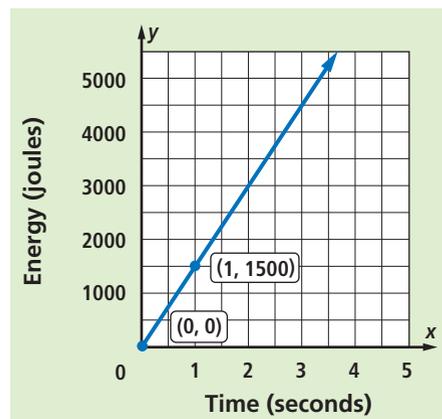
This is the unit rate. This means the microwave uses 1500 joules of energy for every 1 second it runs.



The Origin and $(1, r)$

There are two points of special significance on the graph of a proportional relationship. On the graph in the Example, the point $(0, 0)$ means running the microwave for 0 seconds will use 0 joules of energy. The point $(0, 0)$ appears on the graph of every proportional relationship.

The value of y when $x = 1$ is the numerator of the unit rate, r . Because you know the denominator of the unit rate is 1, you can use this pair of values to easily find the unit rate. On a graph, this point is $(1, r)$, indicating that the unit rate is $\frac{r}{1}$. In the Example the unit rate is 1500 joules per second, and the point $(1, 1500)$ is found on the graph. It is also possible to find the unit rate from any point on the graph by simplifying $\frac{y}{x}$.



as you did in Part b of the Example. For example, the point (22, 33,000) would give the rate $\frac{33,000 \text{ joules}}{22 \text{ seconds}} = \frac{1500 \text{ joules}}{1 \text{ second}} = 1500 \text{ joules per second}$, the unit rate.

Using a Graphing Utility

Activity 2

MATERIALS graphing utility

On September 2, 2010, 1 U.S. dollar was equivalent to about 0.78 euro (€),

$$\text{or } \frac{\text{dollars}}{\text{euro}} = \frac{1}{0.78}.$$

Step 1 Copy and complete the table at the right.

Step 2 For each nonzero pair of values in the chart, write the value as the fraction $\frac{\text{dollars}}{\text{euro}}$. Rewrite each fraction in lowest terms.

Step 3 Find the unit rate to the nearest thousandth.

Step 4 Let y be the dollar cost and x be the price in euro. Write a proportion for this situation. Rewrite your proportion in the form $y =$. Round your coefficient to the nearest thousandth.

Step 5 On the graphing utility, enter your equation from Step 4 in $Y=$.

Step 6 Pick a window that will allow you to trace up through €50. For each value of x given in Parts a–c below, use the **TRACE** feature to find the dollar value of an item that costs x euro. Remember that once you have selected **TRACE**, you can enter a value for x , press **ENTER**, and the corresponding y value will appear on the screen. Round the dollar amount to the nearest cent.

- €15.00
- €28.75
- €42.33

Euro	Dollars
0	?
0.78	1.00
25	?
8.5	?
1000	?

Questions

COVERING THE IDEAS

- Jon rides his bike at an average rate of 12 miles per hour.
 - Copy and complete the table at the right.
 - Graph the pairs of numbers in the table.
 - What is the unit rate?
 - What point on the graph corresponds to this unit rate?
 - What does the point (0, 0) mean in this situation?

Hours	Miles
0	?
2	?
$3\frac{1}{4}$?
$\frac{1}{2}$?

In 2 and 3, simplify and rewrite the equation as $y =$.

2. $\frac{5}{8} = \frac{y}{x}$

3. $\frac{y}{x} = \frac{1.25}{2}$

In 4 and 5, solve each problem using the following steps.

- Write a proportion.
 - Simplify and rewrite the equation as $y =$.
 - Graph your equation from Part b.
 - Explain the meaning of $(0, 0)$ in the context of the situation.
 - Find the value for r , and explain the meaning of $(1, r)$ in the context of the situation.
 - Use the graph to answer the question.
- Alicia can do 3 math problems in two minutes. Assume she completes the rest of the assignment at the same rate. How long will it take her to finish the 42-problem assignment? Use y for the number of minutes and x for number of problems.
 - Larry has a dog-walking business. He charges \$7.50 per hour. During one week last summer, he worked 22.5 hours. How much did he earn that week? Use y for the total amount earned and x for the number of hours worked.

APPLYING THE MATHEMATICS

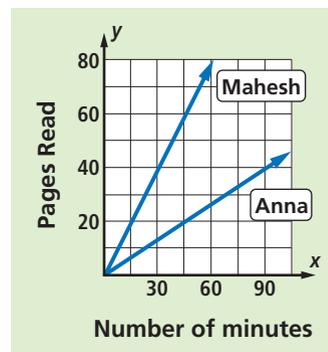
In 6 and 7, simplify and rewrite the equation as $y =$.

6. $\frac{2}{3} = \frac{y}{x}$

7. $\frac{y}{x} = \frac{1}{1.8}$

- The scale on a map of Australia is $1.1 \text{ cm} = 500 \text{ km}$.
 - Write a proportion to show the actual distance y for a distance of x cm on the map.
 - Solve for y . Round to the nearest hundredth.
 - Use a graphing utility to graph the equation. Set the window to include measures on the map up to 5 cm.
 - On the map it is 4.6 cm from Uluru, Ayers Rock, located in central Australia, to Sydney. Use the **TRACE** feature to determine the actual distance from Uluru to Sydney.
 - Explain the meaning of $(0, 0)$ in the context of the situation.
 - Explain the meaning of $(1, y)$ in the context of the situation.

9. Use the graph at the right to answer the following questions.
- What is represented by x ?
 - What is represented by y ?
 - What situation could be described by the graph?
 - What is Mahesh's unit rate?
 - How many pages does Anna read in an hour?
 - Who reads faster?
10. A 360-sheet manuscript weighs 1.62 kg.
- Write an equation for this situation with number of pages as the independent variable and weight as the dependent variable.
 - Graph your equation from Part a.
 - Find the unit rate associated with this situation.
 - Another manuscript printed on the same stock weighs 2052 g. How many sheets are in this manuscript?



QY ANSWER

525 miles