BIG IDEA  Many attributes of a function can be determined by examining its graph.

In companies that mass produce items, it usually costs more per item to produce small quantities than large quantities. However, if too many items are produced, it may require overtime pay or more equipment, and the cost per item may go up again. This kind of situation is pictured in the graph of Example 1.

Example 1  Use the graph at the right to answer the questions.

a. What is the average cost to produce one gear when 250 gears are produced per day?

b. How many gears are being produced per day when the average cost to produce one gear is $1.25?

c. How many gears should the company produce per day if it wants to have the lowest average cost to produce one gear? What is the average cost when the company produces this many gears per day?

Solution

a. The average cost to produce one gear is about $2.75 when 250 gears are produced in a day.

b. When the average cost to produce one gear is about $1.25, the number of gears being produced is either about 2500, or about 2500 gears per day.

c. The cheapest average cost to produce one gear occurs when the company produces 2500 gears per day. This happens when the average cost to produce one gear is $2.00.

A function is increasing on an interval in which the second coordinate increases as the first coordinate increases. A segment connecting two points on an increasing function has a positive slope, so the graph slants up from left to right.

A function is decreasing on an interval in which the second coordinate decreases as the first coordinate increases. A segment connecting two points on a decreasing linear function has negative slope, so the graph slants down from left to right. A constant function does not change values on any interval of the horizontal axis.
Example 2
Consider the average production cost graph of Example 1. On what intervals is the graph increasing? On what intervals is the graph decreasing?

Solution
The graph is sloping up from _____ gears to 3250 gears, so the graph is increasing on the interval _____ < g < 3250.
The graph is sloping down from _____ gears to _____ gears, so the graph is decreasing on the interval _____ < g < _____.

In the situation of Guided Example 2, the intervals where the graph is increasing indicate that increasing the number of gears produced will lead to a higher average production cost. A decreasing interval indicates that increasing the number of gears produced will lead to a lower average production cost. For this company, cost will be minimized if between 1250 and 1750 gears are produced daily.

Example 3
The function \( f \) is graphed below. On what intervals is \( f \) increasing? On what intervals is \( f \) decreasing?

\[
\begin{array}{c}
\text{Solution} \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\end{array}
\]

\( f \) is increasing from -2 to -1 and from 1 to 2.
\( f \) is decreasing from -1 to 1.

It is possible for none of the terms increasing, decreasing, or constant to describe a function on an interval. For example, on the interval from -2 to 0, the function of Example 3 is increasing for part of the interval and decreasing on another part, so we say that \( f \) is neither increasing nor decreasing on that interval.

Linear and Nonlinear Functions
Functions can also be classified as linear or nonlinear on a given interval.
**Example 4**
The function $c$ is graphed at the right. Describe $c$ on the intervals 0 to 1, 1 to 3, 3 to 5, and 5 to 6 as linear or nonlinear and as increasing or decreasing.

**Solution**
From 0 to 1, $c$ is linear and increasing.
From 1 to 3, $c$ is nonlinear and increasing.
From 3 to 5, $c$ is nonlinear and decreasing.
From 5 to 6, $c$ is linear and decreasing.

**Activity**
Fill in the table.

<table>
<thead>
<tr>
<th>Function</th>
<th>Increasing intervals</th>
<th>Decreasing intervals</th>
<th>Linear intervals</th>
<th>Constant intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Graph image]</td>
<td>$x &gt; 0$</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>[Graph image]</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>[Graph image]</td>
<td>?</td>
<td>?</td>
<td>$-2 &lt; x &lt; 0$; $0 &lt; x &lt; 2$; $2 &lt; x &lt; 4$</td>
<td>?</td>
</tr>
<tr>
<td>[Graph image]</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>[Graph image]</td>
<td>?</td>
<td>$0 &lt; x &lt; 2$; $x &gt; 2$</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>
Example 5
Amanda rode her bicycle 4 miles to her friend Sam's house. She rode 12 mph to get there, stayed for half an hour, then rode home at 8 mph. Halfway home she stopped for 5 minutes to talk to a friend. Graph her distance from home \(d\) (in miles), from home as a function of the time \(t\) in minutes since she left home.

Solution Because Amanda started at 12 mph, she rode the 4 miles in 20 minutes. This is pictured by segment \(AB\). She then stayed in the same place for 30 minutes, pictured by \(BC\). After a total of 50 minutes she left for home, traveling 2 miles at 8 mph for a total of 15 minutes pictured by \(CD\). She stayed in the same place for 5 minutes (\(DE\)) and finally rode the last 2 miles in another 15 minutes (\(EF\)).

In Example 5, notice that the increasing interval of the graph represents Amanda’s ride to Sam’s house, the horizontal intervals are the periods when she was not riding, and the decreasing intervals represent her ride home.

Questions

COVERING THE IDEAS

In 1–3, consider the situation of Examples 1 and 2.

1. About what does it cost the company per gear to manufacture 500 gears a day?
2. When the graph is increasing, what does it tell about the production of gears?
3. When the graph is decreasing, what does it tell about the production of gears?
4. Fill in the Blanks A function is increasing on an interval in which, as the value of \(x\) _, the value of \(y\) _.  
5. Fill in the Blank If a linear function is decreasing, it has a _ slope.
6. Describe the graph of a constant function.

In 7 and 8, True or False.

7. It is possible for a nonlinear function to be an increasing function.
8. It is possible for a nonlinear function to be a constant function.
In 9–11, copy and complete the table.

<table>
<thead>
<tr>
<th>Function</th>
<th>Increasing intervals</th>
<th>Decreasing intervals</th>
<th>Linear intervals</th>
<th>Constant intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>11.</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

12. The graph at the right shows Joe's distance from home on his morning jog.
   a. How long was Joe gone?
   b. How far did he jog?

![Graph of Joe's distance from home](image)

**APPLYING THE MATHEMATICS**

In 13–16, sketch a graph to match the verbal description.

13. $f$ is increasing for negative values of $x$ and decreasing for positive values of $x$.

14. $g$ is constant between $-1$ and 1, nonlinear and increasing when $x > 1$, and nonlinear and decreasing when $x < -1$. 

5 Using Algebra to Describe Patterns of Change
15. Lydia gives her dog a bath in the bathtub. When she fills the tub, the water level rises 2 inches per minute for 3 minutes. Then the dog gets in, and the water level rises $2\frac{1}{2}$ inches. After a five-minute bath, the water drains from the tub so the level drops at the rate of $1\frac{1}{3}$ inches per minute. Graph the height $y$ of the water in the tub in inches, based on time $x$ in minutes.

16. An airplane flies 2500 miles from New York City to Los Angeles at an average speed of 500 mph. In the first 15 minutes, the plane climbs to 35,000 feet. It descends at the rate of 1000 feet per minute to land in L.A. Graph the height $y$ of the plane in feet, based on flying time $x$ in hours.