The Akan people of Ghana work with a type of cloth called *adinkra* on which they draw a grid with large rectangles. They then fill each rectangle with a repetitive pattern by sliding the same design along the cloth. Similar techniques can be used for other patterned textiles. A print design can be created by selecting a few distinct square images and tiling them to create a pattern as shown below.

The correspondence between any two images of the same design as in this fabric is called a **slide**, or **translation**. If you visualize a geometric figure on a coordinate grid, a translation is determined by the change in the coordinates necessary to slide one figure onto another.
Horizontal or Vertical Translations

**Activity 1**

Let \( C = (-6, 8) \), \( E = (-7, 0) \), and \( D = (-1, 3) \). Translate \( \triangle CED \) 7 units to the right.

**Step 1** Graph \( \triangle CED \) on a coordinate grid as shown below. \( \triangle CED \) is called the **preimage**.

![Coordinate grid with points C, D, and E marked]

**Step 2** To translate the preimage 7 units to the right, add 7 to each \( x \)-coordinate. The result is a triangle 7 units to the right of \( \triangle CED \). We call this \( \triangle C'E'D' \) (read “triangle C-prime, E-prime, D-prime”). \( \triangle C'E'D' \) is the **translation image** of \( \triangle CED \).

<table>
<thead>
<tr>
<th>Coordinates of Preimage</th>
<th>Translate Preimage 7 Units to the Right</th>
<th>Coordinates of Translation Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C = (-6, 8) )</td>
<td>((-6 + 7, 8))</td>
<td>( C' = (1, 8) )</td>
</tr>
<tr>
<td>( E = (-7, 0) )</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>( D = (-1, 3) )</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

**Step 3** Graph each image point on the same grid. Each image point is 7 units to the right of the preimage point. For instance, \( C' = (1, 8) \) is 7 units to the right of \( C = (-6, 8) \).

In general, if you add \( h \) to each \( x \)-coordinate of the points of a figure, you will get a slide image of the original figure that is \( h \) units to the right when \( h \) is positive. If \( h \) is negative, the image will move to the left. For any preimage point \((x, y)\), the image point after the horizontal translation of \( h \) units is \((x + h, y)\).

(QY)

**a.** To translate \( \triangle CED \) 5 units to the right, add __?__ to the __?__-coordinate.

**b.** To translate \( \triangle CED \) 6 units to the left, add __?__ to the __?__-coordinate.
Activity 2

Translate \( \triangle C'E'D' \) 9 units down.

Step 1 Predict what will happen if you add a particular number to the \( y \)-coordinate.

Step 2 Use your graph from Activity 1. Add -9 to each \( y \)-coordinate of \( \triangle C'E'D' \). We call the image \( \triangle C*E*D* \) (read “triangle C-star, E-star, D-star”). \( C* = (1, -1) \) is the image of \( C' = (1, 8) \) because \( (1, 8 + (-9)) = (1, -1) \).

<table>
<thead>
<tr>
<th>Coordinates of Preimage</th>
<th>Translate Preimage 9 Units Down</th>
<th>Coordinates of Translation Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C' = (1, 8) )</td>
<td>((1, 8 + (-9)))</td>
<td>( C* = _ _ )</td>
</tr>
<tr>
<td>( E' = (0, 0) )</td>
<td>_ _</td>
<td>( E* = _ _ )</td>
</tr>
<tr>
<td>( D' = (6, 3) )</td>
<td>_ _</td>
<td>( D* = _ _ )</td>
</tr>
</tbody>
</table>

Step 3 Graph \( \triangle C*E*D* \).

In general, if you add \( k \) to the second coordinate of all points in a figure, you will slide the figure \( k \) units up when \( k \) is positive. If \( k \) is negative, as it is in Activity 2, then the figure slides down. For any preimage point \((x, y)\), the image point after a vertical translation of \( k \) units is \((x, y + k)\).

**Congruent figures** are figures with the same size and shape. A translation image is always congruent to its preimage. Triangles \( CED \), \( C'E'D' \), and \( C*E*D* \) are all congruent to each other. Using the \( \cong \) symbol for “is congruent to,” you can write the previous sentence as \( \triangle CED \cong \triangle C'E'D' \cong \triangle C*E*D* \).

**Translations That Are Neither Horizontal nor Vertical**

The grid at the right shows \( \triangle CED \) and \( \triangle C*E*D* \). Notice that if you add 7 to the first coordinate and add -9 to the second coordinate of each point of \( \triangle CED \), you get the coordinates of a point on \( \triangle C*E*D* \). Thus, \( \triangle C*E*D* \) is a translation image of \( \triangle CED \).

The rule for this slide can be written as “the image of \((x, y)\) is \((x + 7, y + -9)\).” Because a slide image is congruent to its preimage, you can compute the images of a few special points, like the vertices, and then use a ruler or other tools to complete the image figure.
Translation Image of Any Point

Under a translation $h$ units horizontally and $k$ units vertically, the translation image of any point $(x, y)$ is $(x + h, y + k)$.

**Example**

Let $A = (3, -4)$, $L = (-4, -1)$, and $G = (0, 3)$. Translate $\triangle ALG$ 5 units up and 2 units to the left.

**Solution** The image of $(x, y)$ under this translation is $(x - 2, y + 5)$. So:

$A' = (3 + -2, -4 + 5) = (1, ?)$.  
$L' = (-4 + ?, -1 + ?) = (?, ?)$.  
$G' = (?, ? + 3, 3 + ?) = (?, ?)$

**Questions**

**COVERING THE IDEAS**

In 1–4, complete the chart. In each case, consider the preimage point $(x, y)$.

<table>
<thead>
<tr>
<th>If</th>
<th>Then the Figure Slides</th>
<th>Coordinates of Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 4 is added to the $x$-coordinate of every point on the figure,</td>
<td>(?).</td>
<td>$(x + ?, y)$</td>
</tr>
<tr>
<td>2. $?$ is added to the $?_{\text{coordinate of}}$ every point on the figure,</td>
<td>7.3 units up.</td>
<td>$(x, y + ?)$</td>
</tr>
<tr>
<td>3. $?$ is added to the $?_{\text{coordinate of}}$ every point on the figure,</td>
<td>(?).</td>
<td>$(x, y + -11\frac{3}{5})$</td>
</tr>
<tr>
<td>4. $?$ is added to the $?_{\text{coordinate of}}$ every point on the figure,</td>
<td>(?).</td>
<td>$(x + -5, y)$</td>
</tr>
</tbody>
</table>

5. Another name for translation is __?__.  
6. When you change coordinates of points of a figure to get another figure, the original figure is called the __?__ and the resulting figure is called its __?__.

In 7 and 8, copy the grid at the right. Then graph $\triangle HOG$ and its image under the translation that is described.

7. The image of $(x, y)$ is $(x + 3, y)$.
8. The image of $(x, y)$ is $(x + -3, y + -2)$.  

Translations 6
In 9 and 10, tell what happens to the graph of a figure when
9. $k$ is added to the second coordinate and $k$ is negative.
10. $h$ is added to the first coordinate and $h$ is positive.
11. Tell whether this statement is always, sometimes but not always, or never true:
   A figure and its translation image are congruent.
12. If $(x, y)$ is a preimage point, explain what transformation yields
    $(x + 100, y + -500)$ as the image point.

**APPLYING THE MATHEMATICS**

13. Polygon $ABCDEFGH$ outlines a top view of a school building.

   ![Diagram of Polygon A'B'C'D'E'F'G'H']

   The architect wishes to send this outline via e-mail to a builder. To avoid negative numbers, the architect slides the graph so that the image of point $A$ is the origin.

   ![Diagram showing the shift]  

   What are the coordinates of $B'$, $C'$, $D'$, $E'$, $F'$, $G'$, and $H'$?
14. **a.** Draw quadrilateral $PQRS$ with $P = (0, 0)$, $Q = (6, 0)$, $R = (6, 2)$, and $S = (0, 4)$.

**b.** On the same axes, draw the image of $PQRS$ when 2 is subtracted from each *first* coordinate and 4 is subtracted from each *second* coordinate.

**c.** How are the preimage and image related?

15. $\triangle A'B'C'$ is a slide image of $\triangle ABC$. $A = (0, 0)$, $B = \left(\frac{1}{2}, 0\right)$, $C = \left(0, \frac{1}{3}\right)$, and $C' = \left(\frac{1}{4}, \frac{1}{5}\right)$. What are the coordinates of $A'$ and $B'$?

16. Draw three points and their images under the transformation in which the image of $(x, y)$ is $(x + 4, y - 5)$.

17. Suppose $\triangle Q'R'S'$ is the image of $\triangle QRS$ as a result of the translation $(x + 4.7, y - 5.3)$. How would you translate point $(a, b)$ on $\triangle Q'R'S'$ back to $\triangle QRS$?

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**QY ANSWER**

a. 5; $x$

b. –6; $x$