

Lesson

14-3

Rotations

Vocabulary

transformation
 rotation
 center of rotation
 magnitude
 rotation image

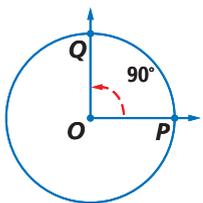
► **BIG IDEA** The rotation image of a figure about a point can be found by rotating key points of the figure about the point.

So far in this chapter, you have seen two ways to change or *transform* a figure to a congruent figure—by a translation and by a reflection. A **transformation** is a one-to-one correspondence between a first set of points (the preimage) and a second set of points (its image). This lesson is about a third transformation that yields congruent figures, the *rotation*.

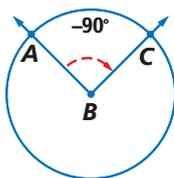
Understanding Rotations

A **rotation** in a plane is the turn of a figure around a point called the **center of rotation**. The **magnitude** of a rotation measures how much a figure has turned and in what direction. The direction may be clockwise or counterclockwise. It is tradition in mathematics that *counterclockwise* rotations are given positive magnitudes. You can use a protractor to help measure magnitudes of rotation.

Two quarter-turns pictured below are in opposite directions. The counterclockwise turn at the left has a magnitude of 90° . The clockwise turn at the right has a magnitude of -90° .

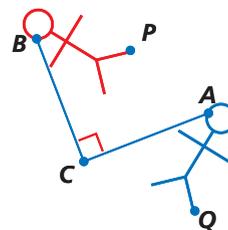


counterclockwise turn
 Q is the image of P
 (positive magnitude)



clockwise turn
 C is the image of A
 (negative magnitude)

At the right, the blue stick figure has been rotated to get the red stick figure. The center of the rotation is C. Notice that $CA = CB$. Since $m\angle BCA = 90^\circ$ and the rotation is counterclockwise, the magnitude of the rotation is 90° .



► QY1

What is the measure of $\angle PCQ$?

STOP QY1

Example

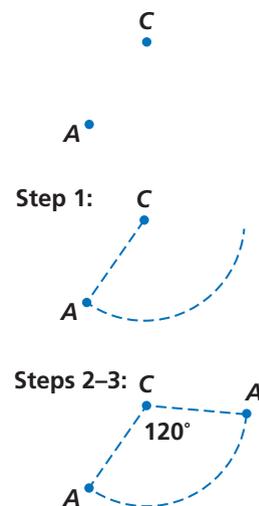
In the figure at the right, find the image of point A under a rotation of 120° about point C .

Solution

Step 1 Draw a circle (lightly) with point C as the center and containing point A . Because A is rotated around C , the image must be on this circle.

Step 2 Since 120° is positive, the rotation is counterclockwise. Measure an angle of 120° with \overrightarrow{CA} as one side.

Step 3 The intersection of the other side of the angle and the circle is the desired image A' .



When a figure is rotated, each image point is the same distance from the center as its preimage.

Activity 1

MATERIALS thin paper such as tracing paper, a pencil, a protractor, and a ruler

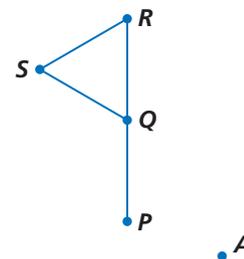
Find the image of the flag $PQRS$ when rotated 90° about point A .

Step 1 Trace the preimage and point A on a piece of thin paper.

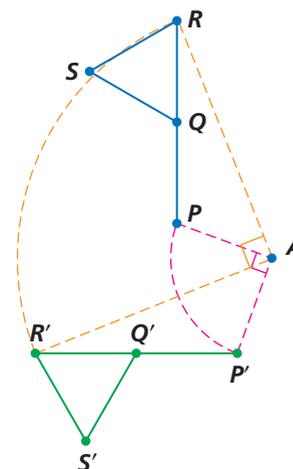
Step 2 Predict and draw where the rotation image will be.

Step 3 With a second sheet of thin paper, trace the preimage and A again. Place the second tracing over the first. Holding the tip of your pencil firmly at A , rotate the bottom piece of thin paper 90° . Then draw the rotated image.

Step 4 Compare your prediction with the rotated image. How accurate were your predictions?



You can check your work in Activity 1 by measuring. Your image and preimage should look like the figure at the right. Consider point P on the preimage and point P' on the rotation image. Notice that the preimage point rotates on a circle with center A and radius \overline{PA} through a turn of 90° . Thus, $PA = P'A$ and $\overline{PA} \perp \overline{P'A}$. You should see similar relationships between each preimage point and its image.



STOP QY2

This leads to the definition of rotation image.

Definition of Rotation Image

Let A and P be points. Under a rotation with center A and magnitude x° , the **rotation image** P' of P satisfies $\overline{PA} = \overline{P'A}$ and $m\angle PAP' = x^\circ$. If x is positive, measure from \overrightarrow{AP} to $\overrightarrow{AP'}$ in a counterclockwise direction. If x is negative, measure from \overrightarrow{AP} to $\overrightarrow{AP'}$ in a clockwise direction.

STOP QY3

A rotation image is always congruent to its preimage, so in Activity 1, $PQRS \cong P'Q'R'S'$.

► QY2

Use your drawing from Activity 1 and draw \overline{RA} and $\overline{R'A}$. Verify by measuring that $RA = R'A$ and $\overline{RA} \perp \overline{R'A}$.

► QY3

A rotation of -90° rotates a figure in a ? direction.

Activity 2

MATERIALS colored pencils, a ruler, a protractor, and a compass

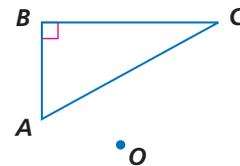
Rotate $\triangle ABC$ by -40° about point O .

Step 1 Trace $\triangle ABC$ and point O so that point O is near the center of your paper.

- Step 2**
- Draw \overline{OA} .
 - Use a protractor to place point D so that $m\angle AOD = 40^\circ$ with point D clockwise from point A .
 - Use a compass to draw a circle with center O and radius \overline{OA} .
 - The rotation image A' is the point of intersection of \overrightarrow{OD} and the circle. Mark A' in a different color.

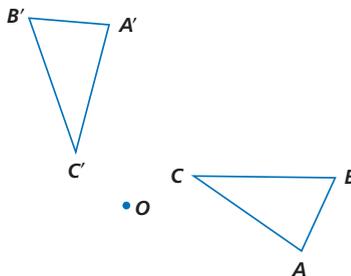
Step 3 Repeat Step 2 with points B and C to find the corresponding image points B' and C' .

Step 4 Connect A' , B' , and C' to form $\triangle A'B'C'$.

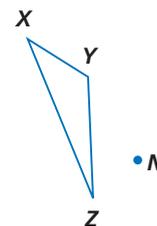
**Questions****COVERING THE IDEAS**

- Name three types of transformations that yield congruent figures.
- A rotation is described by its ? and ?.
- The distance from the center of a rotation to a preimage point is equal to the distance from ? to ?.

4. $\triangle A'B'C'$ is the image of $\triangle ABC$ under a rotation of 110° around O .
- Name three angles that measure 110° . (The angles may not be drawn.)
 - If $AO = 6$, find $A'O$.

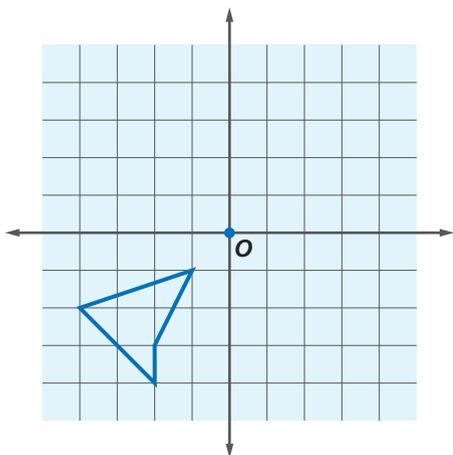


5. Trace the figure at the right. Draw a rotation of $\triangle XYZ$ -80° about point N .

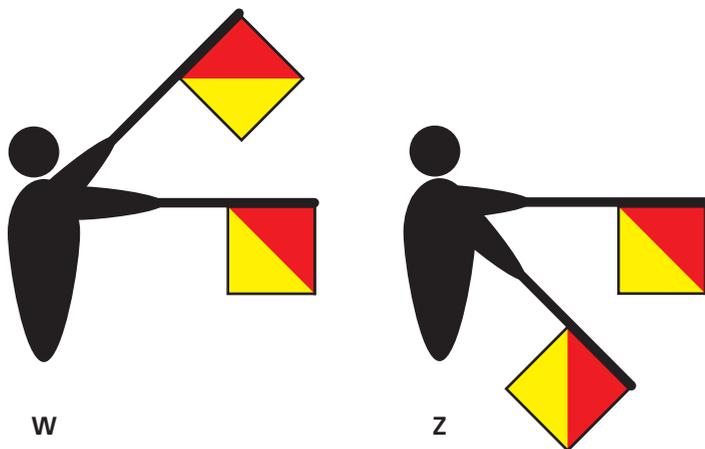


APPLYING THE MATHEMATICS

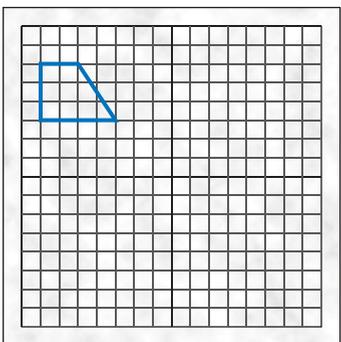
6. Rotate the figure below -90° around point O .



7. Flag semaphore is a way to communicate visually at a distance. The positions signaling “W” and “Z” are shown below. Describe the center and magnitude of the rotation that maps the flags for W onto the flags for Z.



8. If a figure is rotated x° ; how many degrees must its image be rotated to return to its original position?
9. a. Trace the trapezoid and the coordinate axes below on a piece of thin paper. Then rotate the trapezoid 180° about the origin.

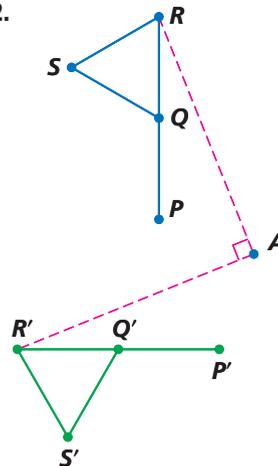


- b. Trace the preimage from Part a and the axes onto a second sheet of thin paper. Reflect the preimage across the x -axis and then reflect the image across the y -axis.
- c. Compare the final images from Parts a and b. Write what you observe.
10. Draw accurate pictures and answer the questions.
- Under a rotation of 90° around $(0, 0)$, what are the coordinates of the image of $(-5, 2)$?
 - Repeat Part a for the point $(3, 4)$.
 - Repeat Part a for the point $(-1, 3)$.
 - Use Parts a–c or try more points to obtain a formula for the image of (x, y) under a rotation of 90° about $(0, 0)$.
11. A combination lock has 36 numbers arranged clockwise and evenly spaced in a circle. Suppose the number 14 is currently at the top.
- What is the magnitude of the clockwise rotation that will place 21 at the top?
 - What is the magnitude of the counterclockwise rotation that will place 21 at the top?
 - What number on the lock is 30° away from 21? -50° away?
12. Through how many degrees does the indicated hand on a clock rotate in the given time?
- hour hand, 4:00 A.M. to 9:00 A.M.
 - minute hand, 3:51 P.M. to 4:33 P.M.
 - second hand, in 1 second
 - hour hand, in 3 hours

QY ANSWERS

1. 90°

2.



3. clockwise