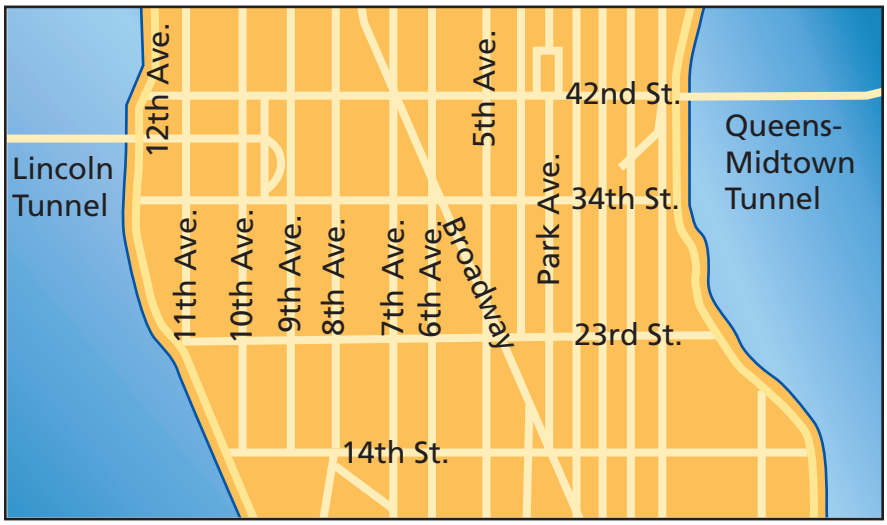
Lesson

14-5 Angles and Lines

Vocabulary

opposite rays
straight angle
linear pair
vertical angles

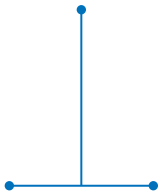
BIG IDEA When two lines intersect, pairs of the four angles formed are either supplementary linear pairs or congruent vertical angles.



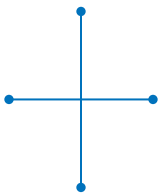
In every town or city, there are many examples of intersecting line segments. Each intersection of line segments represents where two roads intersect. A map of part of Manhattan in New York City is shown above.

Activity 1

On the map, find an example of intersecting line segments similar to each of the following. Name the streets that intersect.

1.


2.


3.


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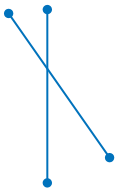
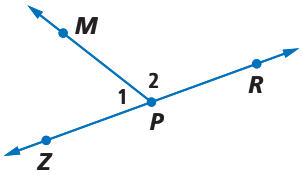


From the map, you can see that there are many ways for two line segments or lines to intersect. In this lesson, you will see names for these figures and angles as well as their special properties.

Linear Pairs

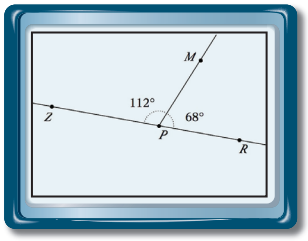
In the figure at the right, you can think of one street stopping at another. $\overrightarrow{PZ}$ and $\overrightarrow{PR}$ are called **opposite rays** because they have the same endpoint and their union is line $\overleftrightarrow{ZR}$. $\angle ZPR$ is called a **straight angle**.



$\angle MPR$ and $\angle MPZ$ are called a **linear pair** because they have a common side $\overrightarrow{PM}$, and their noncommon sides, $\overrightarrow{PR}$ and $\overrightarrow{PZ}$, are opposite rays.

Activity 2

This activity can be done using a Dynamic Geometry System (DGS) or a protractor.



Step 1 Construct a diagram like the one above.

Step 2 If you are using a DGS, measure $\angle ZPM$ and $\angle RPM$ and find their sum. Drag M to move $\overrightarrow{PM}$. You should see that although $\angle ZPM$ and $\angle RPM$ vary in measure, the sum of their measures does not change. If you are using a protractor, draw two more diagrams like the one above, measure the resulting angles, and find their sum.

In Activity 2, you should notice that the sum of the angles in a linear pair is 180°. Since $\angle ZPR$ is a straight angle and the measure of a straight angle is 180°, this should not be surprising.

STOP **QY1**

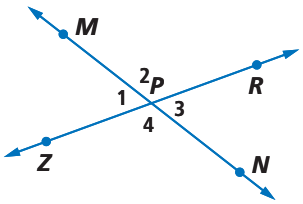
▶ QY1

a. One angle of a linear pair measures 12°. What is the measure of the other angle?

b. If one angle of a linear pair measures a°, what is the measure of the other?

Vertical Angles

When two lines intersect, they form a figure like the one below. Four linear pairs are formed.



STOP **QY2**

▶ QY2

Name the four linear pairs in the figure.

In the figure above, $\angle MPR$ and $\angle NPZ$ are called *vertical angles*. Two angles are **vertical angles** when the opposite rays of the sides of one angle are the sides of the other angle.

STOP **QY3**

▶ QY3

Name the second pair of vertical angles.

Because we know that ∠1 and ∠2 are a linear pair, the measure of ∠1 must be 180 − m∠2. Similarly, since ∠2 and ∠3 are a linear pair, the measure of ∠3 must be 180 − m∠2. So, ∠1 and ∠3 have the same measure. That is, m∠1 = m∠3. Similarly, m∠2 = m∠4. This argument shows:

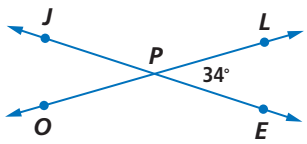
Vertical Angles Theorem

Vertical angles have the same measure.

GUIDED

Example 1

If m∠*LPE* = 34°, find m∠*EPO*, m∠*JPO*, and m∠*LPJ*.



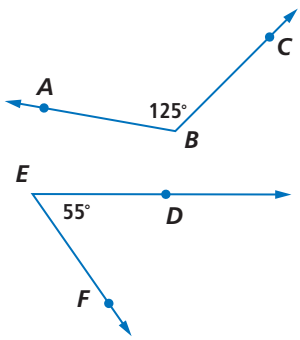
Solution ∠*LPE* and ∠*JPO* are vertical angles. Vertical angles have the same measure, so m∠JPO = __?__.

∠*LPE* and ∠*LPJ* form a linear pair, so m∠*LPJ* = 180° − 34°.
Thus, m∠LPJ = __?__.

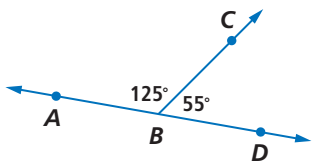
∠*LPE* and ∠*EPO* form a linear pair, so m∠EPO = __?__.

What Is the Difference between Supplementary Angles and Linear Pairs?

Recall that supplementary angles are two angles whose measures add to 180°. Supplementary angles do not have to form a linear pair because they do not have to be adjacent angles. For example, any two angles with measures 55° and 125° are supplementary no matter where the angles are located, because 55° + 125° = 180°. Linear pairs must be both supplementary and adjacent.



∠*ABC* and ∠*DEF* are supplementary angles but not a linear pair.



∠*ABC* and ∠*CBD* are supplementary angles and a linear pair.

Example 2

$\angle RWC$ and $\angle XYZ$ are supplementary. If $m\angle RWC = 4a - 42$ and $m\angle XYZ = 18 + 8a$, find the measures of the two angles.

Solution First find a.

$m\angle RWC + m\angle XYZ$	$= 180°$	Measures of supplementary angles add to 180°.
$(4a - 42) + (18 + 8a)$	$= 180$	Substitution
$(4a + 8a) + (-42 + 18)$	$= 180$	Commutative and Associative Properties of Addition
$12a - 24$	$= 180$	Arithmetic
$12a$	$= 204$	Add 24 to both sides.
a	$= 17$	Divide both sides by 12.

Now substitute 17 for a to find the angle measures.

$m\angle RWC = 4a - 42 = 4(17) - 42 = 26$, so $m\angle RWC = 26°$.

$m\angle XYZ = 18 + 8a = 18 + 8(17) = 154$, so $m\angle XYZ = 154°$.

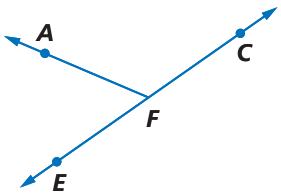
Check $26° + 154° = 180°$

Questions

COVERING THE IDEAS

1. **Fill in the Blank**
   a. The sum of the measures of the angles of a linear pair is __?__.
   b. Vertical angles have __?__ measure.
2. a. Draw two intersecting lines.
   b. Identify the pairs of vertical angles.
3. Draw an example of supplementary angles that are *not* a linear pair. Label each angle with its measure.

In 4–6, **Fill in the Blank.**
Refer to the diagram at the right. F is a point on $\overleftrightarrow{EC}$.



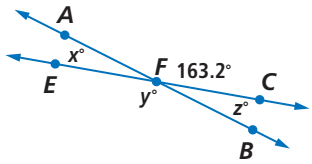
4. $\overrightarrow{FC}$ and $\overrightarrow{FE}$ are __?__ rays.
5. $\angle AFC$ and $\angle AFE$ are a(n) __?__ pair.
6. $m\angle AFC + m\angle AFE =$ __?__

In 7–10, **True or False. Justify your answer.**

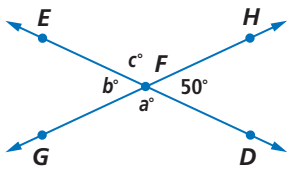
7. If two angles are supplementary, the sum of their measures must be 180°.
8. If two angle measures add to 180°, the angles are supplementary.
9. If two angles are supplementary, then they are a linear pair.
10. If two angles are a linear pair, then they are supplementary.

In 11 and 12, find the three unknown angle measures without using a protractor.

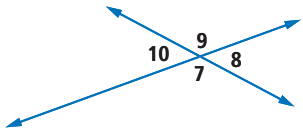
11.



12.

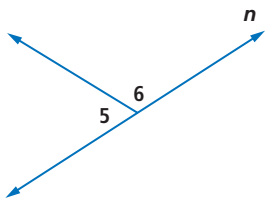


13. Refer to the diagram at the right. If m∠7 = k, find m∠8, m∠9, and m∠10.

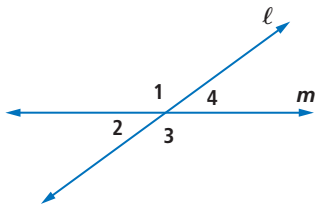


APPLYING THE MATHEMATICS

14. Angles 5 and 6 form a linear pair. If m∠5 = $(57 + \frac{1}{2}x)°$, and m∠6 = $(3x - 17)°$, find their measures.



15. Lines ℓ and m intersect, forming the numbered angles. If m∠3 = 5 m∠4, find the measures of all four angles.



In 16 and 17, True or False. Justify your answer.

16. If one angle of a linear pair is an obtuse angle, then the other angle is obtuse.

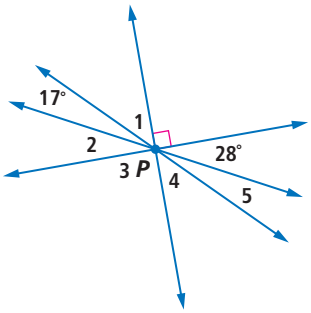
17. If one of two vertical angles is acute, then the other angle is acute.

18. Here is a map of part of Memorial Park in Houston, Texas. Notice that eight angles are numbered.

a. Which pairs of numbered angles form linear pairs?

b. Which pairs of numbered angles form vertical angles?

19. Four lines all contain point *P*, as shown below. Find the measure of each numbered angle.



EXPLORATION

20. Entrance ramps onto expressways usually form very small angles with the expressway. This is bad for visibility and may be quite dangerous. Why aren't entrance ramps perpendicular to expressways?

QY ANSWERS

1. a. 168°

b. $(180 - a)°$

2. $\angle 1$ and $\angle 2$, $\angle 2$ and $\angle 3$, $\angle 3$ and $\angle 4$, $\angle 1$ and $\angle 4$

3. $\angle MPZ$ and $\angle RPN$