Lesson

14-6 Angles and Parallel Lines

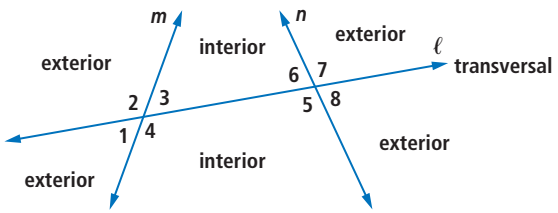
BIG IDEA When two lines are intersected by a third line, eight angles are formed. If the lines are parallel, then the eight angles either have the same measure or are supplementary.

Vocabulary

transversal
exterior angles
interior angles
exterior angles on the same side of the transversal, same-side exterior angles
interior angles on the same side of the transversal, same-side interior angles
alternate exterior angles
alternate interior angles
corresponding angles
||, is parallel to

Angles Formed by Two Lines and a Transversal

Consider a line that intersects two other lines. A line that intersects at least two others is called a **transversal**. Eight angles are formed when a transversal intersects two lines.



In the figure above, ℓ is a transversal intersecting lines m and n. Angles 1, 2, 7, and 8 are called **exterior angles** because they are *outside* lines m and n. Angles 3, 4, 5, and 6 are called **interior angles** because they are *between* lines m and n.

Activity 1

Refer to the figure above. Pairs of angles—one at each intersection—have different names depending on their positions in the figure. The names for one pair of each type of angle is given. Name the other pair.

Step 1 **Exterior angles on the same side of the transversal**:
1 and 8, __?__ and __?__
Interior angles on the same side of the transversal:
4 and 5, __?__ and __?__
To save some writing, we sometimes refer to these angles as **same-side exterior angles** and **same-side interior angles**.

Step 2 The word *alternate* means that one angle is on one side of the transversal, and the other angle is on the other side.
Alternate exterior angles: 1 and 7, __?__ and __?__
Alternate interior angles: 4 and 6, __?__ and __?__

Step 3 Other pairs are named by their relation to each other. The word *corresponding* means that the angles are in the same relative position in the intersections. There are four pairs of corresponding angles.
Corresponding angles: 1 and 5, 2 and __?__, 7 and __?__, __?__ and __?__

When the two lines intersected by a transversal are parallel, the eight angles formed have interesting properties.

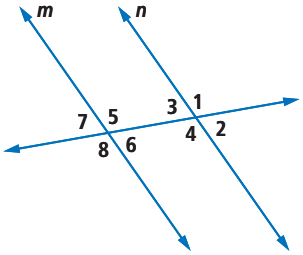
Activity 2

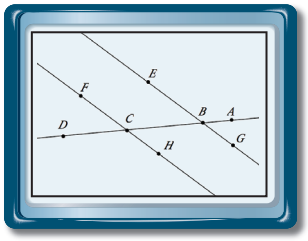
You can complete this activity using either a DGS or a protractor.

Step 1 Draw two parallel lines.

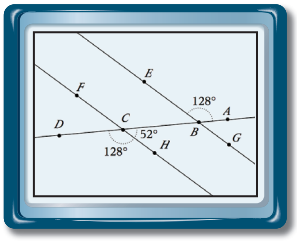
Step 2 Draw a transversal that is *not* perpendicular to the parallel lines. On a DGS, construct the points where the transversal intersects the parallel lines.

Step 3 If you are drawing the diagram by hand, your drawing should look like the picture below at the left. If you are using a DGS, names of points may appear automatically or you may need to name them.





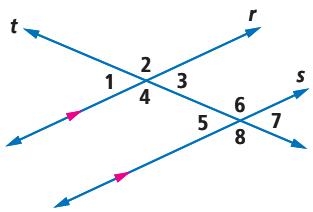
Step 4 If you are using a DGS, you will need to have two points on each line to measure angles. If you are drawing by hand, label the angles on the parallel lines as shown above at the left. Find the measures of all eight angles formed. The screen shot at the right shows what your DGS may look like as you find these measures.



Step 5 Complete the table. If you are using a DGS, drag a point on one of the parallel lines and see how the angles change. Which angle measures remain equal to each other? If you are using a drawing with ruler and pencil, measure the angles with a protractor and compare your measures with those of others in your class.

Location of Angle Pairs	Names of Angles from Labeled Drawing	Measures of These Angles	Relationship (Congruent or Supplementary)
Alternate interior angles	Pair 1 ∠3 and ∠6 Pair 2 ___?___ and ___?___	___?___ and ___?___ ___?___ and ___?___	?
Alternate exterior angles	Pair 1 ___?___ and ___?___ Pair 2 ___?___ and ___?___	___?___ and ___?___ ___?___ and ___?___	?
Same-side exterior angles	Pair 1 ___?___ and ___?___ Pair 2 ___?___ and ___?___	___?___ and ___?___ ___?___ and ___?___	?
Same-side interior angles	Pair 1 ___?___ and ___?___ Pair 2 ___?___ and ___?___	___?___ and ___?___ ___?___ and ___?___	?
Corresponding angles	Pair 1 ___?___ and ___?___ Pair 2 ___?___ and ___?___ Pair 3 ___?___ and ___?___ Pair 4 ___?___ and ___?___	___?___ and ___?___ ___?___ and ___?___ ___?___ and ___?___ ___?___ and ___?___	?

Angles Formed by Parallel Lines and a Transversal



At the right, lines r and s are parallel. We write this as $r \parallel s$. The symbol $\parallel$ is read "**is parallel to**." Line t is the transversal.

The properties of measures of angles formed by parallel lines and a transversal are usually written as if-then statements.

1. If parallel lines are cut by a transversal, then corresponding angles have the same measure.

   If $r \parallel s$, then $m\angle 1 = m\angle 5$, $m\angle 2 = m\angle 6$, $m\angle 3 = m\angle 7$, and $m\angle 4 = m\angle 8$.
2. If parallel lines are cut by a transversal, then alternate interior angles have the same measure.

   If $r \parallel s$, then $m\angle 3 = m\angle 5$ and $m\angle 4 = m\angle 6$.
3. If parallel lines are cut by a transversal, then alternate exterior angles have the same measure.

   If $r \parallel s$, then $m\angle 1 = m\angle 7$ and $m\angle 2 = m\angle 8$.

4. If parallel lines are cut by a transversal, then same-side interior angles are supplementary.

   If $r \| s$, then $m\angle 4 + m\angle 5 = 180°$ and $m\angle 3 + m\angle 6 = 180°$.

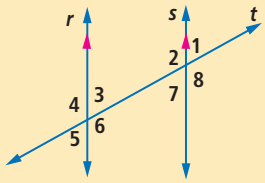
5. If parallel lines are cut by a transversal, then same-side exterior angles are supplementary.

   If $r \| s$, then $m\angle 1 + m\angle 8 = 180°$ and $m\angle 2 + m\angle 7 = 180°$.

STOP QY1

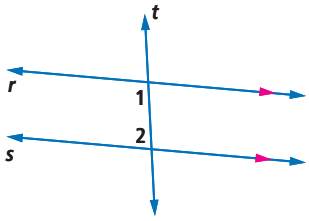
QY1

Find the measure of each angle given that $r \| s$ and $m\angle 1 = 56°$.



Example

Lines r and s are parallel.
If the measure of $\angle 1$ is 15° larger than the measure of $\angle 2$, find $m\angle 1$ and $m\angle 2$.



Solution Ask yourself: What kind of angles are $\angle 1$ and $\angle 2$?
$\angle 1$ and $\angle 2$ are same side interior angles, so $m\angle 1 + m\angle 2 = 180°$. Also $m\angle 1 = m\angle 2 + 15°$.

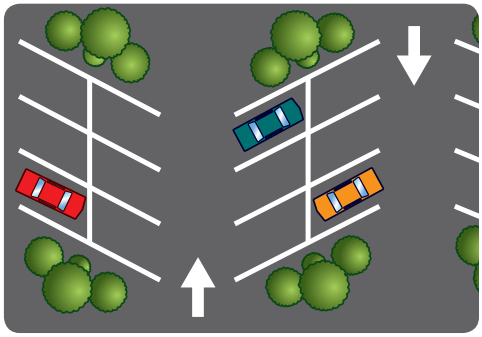
$m\angle 1 + m\angle 2$	$= 180°$	Same-side interior angles are supplementary.
$m\angle 2 + 15 + m\angle 2$	$= 180°$	Substitution
$2(m\angle 2)$	$= 165°$	Add –15 to both sides.
$m\angle 2$	$= 82.5°$	Divide both sides by 2.
$m\angle 1$	$= 180° - 82.5° = 97.5°$	

Check Is $m\angle 1$ larger than $m\angle 2$ by 15°? Yes.
Are they supplementary angles? Yes.

Converses of Properties of Parallel Lines

Although converses of if-then statements are not necessarily true, the converse of each property of angles formed by parallel lines intersected by a transversal is true. Here are the converses.

1. If two lines cut by a transversal have corresponding angles with the same measure, then the lines are parallel.
2. If two lines cut by a transversal have alternate interior angles with the same measure, then the lines are parallel.
3. If two lines cut by a transversal have alternate exterior angles with the same measure, then the lines are parallel.
4. If two lines cut by a transversal have same-side interior angles that are supplementary, then the lines are parallel.
5. If two lines cut by a transversal have same-side exterior angles that are supplementary, then the lines are parallel.



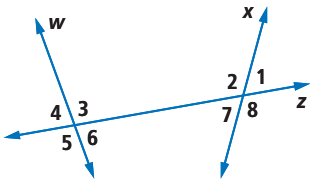
You can see parallel lines cut by a transversal in many parking lots.

STOP QY2

Questions

COVERING THE IDEAS

In 1–11, refer to the diagram at the right.

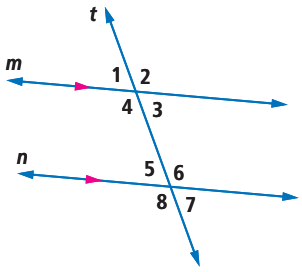


1. Name the transversal.
2. Name all pairs of alternate interior angles.
3. Name all pairs of same-side interior angles.
4. Name all pairs of alternate exterior angles.
5. Name all pairs of same-side exterior angles.
6. Name all pairs of corresponding angles.
7. Name all pairs of vertical angles.
8. Name all linear pairs.
9. If $m\angle 4 = 63°$, find $m\angle 5$.
10. If $m\angle 2 = 129°$, find $m\angle 1$.
11. If $m\angle 6 = 71.3°$, find $m\angle 3$, $m\angle 4$, and $m\angle 5$.

In 12–14, Fill in the Blank.

12. If two parallel lines are cut by a transversal, corresponding angles have ___?___ measure.
13. If same-side interior angles are ___?___, then lines cut by the transversal are parallel.
14. If two parallel lines are cut by a transversal, same-side exterior angles are ___?___.

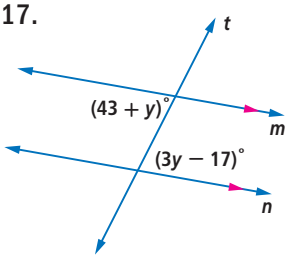
In 15 and 16, use the diagram at the right in which $m \parallel n$. Find the measure of the other seven angles from the given information.



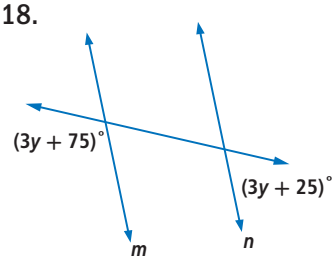
15. $m\angle 3 = 62°$
16. $m\angle 8 = 109°$

In 17 and 18, $m \parallel n$. Find the value of the variable.

17.



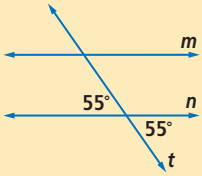
18.



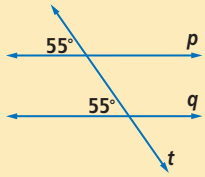
QY2

For Parts a and b, use the angle measures shown in the diagrams. Can you conclude that the lines intersected by *t* are parallel? Give a reason for your answer.

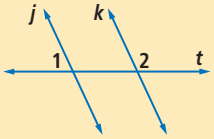
a.



b.

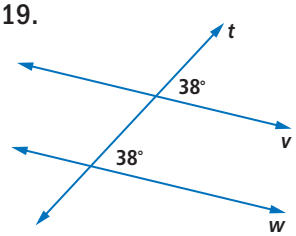


c. In the figure below, if $m\angle 2$ is 100°, what would $m\angle 1$ have to be for *j* to be parallel to *k*?

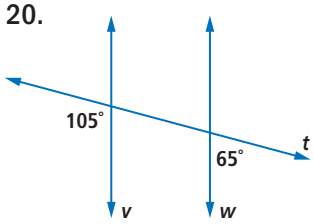


In 19 and 20, is $v \parallel w$? Justify your answer.

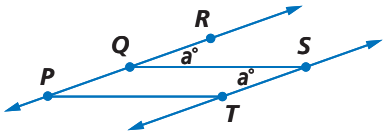
19.



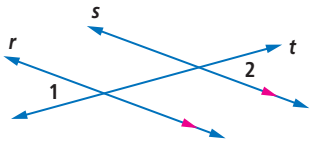
20.



21. What can you conclude from the picture at the right? Explain why you made this conclusion.



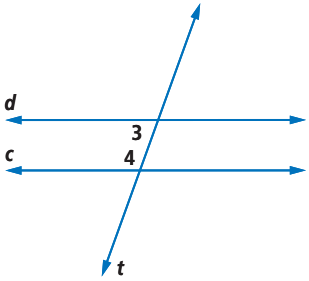
22. Refer to the drawing below. Suppose $r \parallel s$, $m\angle 1 = (8v + 10)°$, and $m\angle 2 = (50 - 2v)°$.



a. What is the relationship between $\angle 1$ and $\angle 2$?

b. Find $m\angle 1$ and $m\angle 2$.

23. Refer to the drawing at the right. Suppose $m\angle 4 = (x + 27)°$ and $m\angle 3 = (4x - 22)°$.

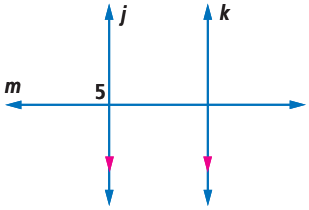


a. What kind of angles are $\angle 3$ and $\angle 4$?

b. Find the value of x that would make $c \parallel d$.

APPLYING THE MATHEMATICS

24. **True or False** Use the figure at the right. If $m\angle 5 = 90°$ and $j \parallel k$, then the measure of every other angle is 90°.



25. Consider this statement:
If two lines are cut by a transversal and a pair of corresponding angles each measure 82°, then the lines are parallel.

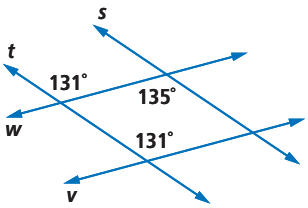
a. Write the converse.

b. Is the converse true or false? If false, provide a counterexample.

26. **a.** Draw $d \parallel m$ with transversal ℓ. Place $\angle 1$, $\angle 2$, and $\angle 3$ in the drawing so that the following criteria are met:
$\angle 1$ and $\angle 2$ are corresponding angles.
$\angle 1$ and $\angle 3$ are alternate exterior angles.

b. What is true about the three angles?

27. Use the angle measures given in the diagram at the right.



a. Which lines are parallel?

b. How do you know the lines are parallel?

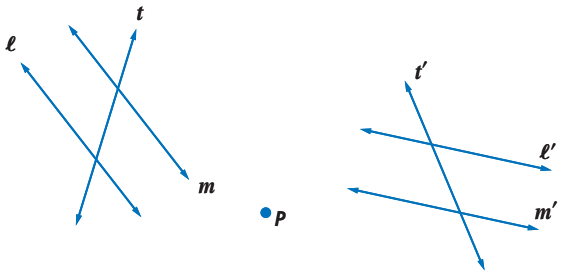
c. Which lines are not parallel?

In the diagram below, consider this partial street map of San Francisco. Turk Street intersects Market Street at an angle of 39°. Golden Gate Avenue intersects 6th Street at an angle of 129°. 6th Street and 7th Street are parallel.

28. If Turk Street and Golden Gate Avenue are parallel, at what angle does Market Street intersect 6th Street?

29. If 7th Street were extended to intersect Golden Gate Avenue, what is the measure of the smallest angle it would form?

30. In the diagram, ℓ and m are parallel and cut by transversal t. ℓ', m', and t' are the rotation image of ℓ, m and t around P.



a. Copy the diagram.
b. Label two angles of equal measure in the preimage $\angle 1$ and $\angle 2$. Do not use vertical angles.
c. Identify the image of $\angle 1$. Label it $\angle 3$.
d. Identify the image of $\angle 2$. Label it $\angle 4$.
e. What is true about $m\angle 3$ and $m\angle 4$?
f. What must be true about ℓ' and m'? Justify your answer.

31. **True or False** In a translation, if lines are parallel in the preimage, they will be parallel in the image.

QY ANSWERS

1. $m\angle 2 = m\angle 4 = m\angle 6 = m\angle 8 = 124°$
$m\angle 1 = m\angle 3 = m\angle 5 = m\angle 7 = 56°$

2. **a.** No; vertical angles always have equal measure.

b. Yes; if corresponding angles have equal measure, then lines are parallel.

c. 80°