BIG IDEA The size-change image of a figure can be found by multiplying the coordinates of each point on a figure by a fixed nonzero number.

In Lesson 14-1, you learned that adding fixed numbers to each of the coordinates of a figure has the effect of translating the figure. What happens when you multiply the coordinates by a fixed number?

**Activity 1**

**MATERIALS** ruler, protractor, graph paper

**Step 1** Make a copy of the figure below. This figure is the preimage.

![Graph showing a preimage of a figure](image)

**Step 2**

a. On the same grid, create an image by multiplying each coordinate of the named points by 3. Label these new points $A'$, $B'$, and so on. For example, because $B = (1, 5)$, $B' = (3\cdot1, 3\cdot5) = (3, 15)$.

b. Describe the shape of the image.
Step 3  The lines $AA'$, $BB'$, $CC'$, and so on, through points and their images are **concurrent**. This means that they all have a point in common. This point is called the **point of concurrency**. What is the point of concurrency of these three lines?

In Steps 4–6, answer the question. Support your answer with an example from the diagram.

Step 4  How are the lengths in the preimage related to the lengths in the image?

Step 5  How are the measures of the angles in the preimage related to the measures of the angles in the image?

Step 6  How are the slopes of segments in the preimage related to slopes of the corresponding segments in the image?

You should have found in this Activity that you created a figure whose lengths were 3 times as long, but otherwise all of the other qualities of the shape of the figure remained the same. In other words, the figure looked exactly the same, just larger.

This transformation is called a **size change** or **dilation** of magnitude 3. If you multiply the coordinates of all points on the plane by the same number (not just 3), you will produce an image under a size change. In this size change, the origin $(0, 0)$ is on the line containing any point and its image. We call $(0, 0)$ the **center** of the size change.

**Size-Change Image of Any Point**

Under a size change of magnitude $k \neq 0$, the size-change image of any point $(x, y)$ is $(kx, ky)$.

In Activity 1, the size-change image was larger than the original figure. For what positive value(s) of $k$ would the size-change image be smaller than the original figure?
Example 1

\( \triangle LMN \) is shown at the right. Graph its image under a size change of magnitude \( \frac{3}{4} \).

Solution

Since \( L = (-8, -12) \),
\( L' = \left( \frac{3}{4} \cdot -8, \frac{3}{4} \cdot -12 \right) \)
\( = (-6, -9) \).

Since \( M = (12, \_\_\_) \),
\( M' = \left( \frac{3}{4} \cdot 12, \_\_ \cdot \_\_ \right) \)
\( = (\_\_, \_\_). \)

Since \( N = (\_\_, \_\_) \),
\( N' = \left( \_\_ \cdot \_\_, \_\_ \cdot \_\_ \right) \)
\( = (\_\_, \_\_). \)

Properties of Size Changes

Size changes can occur without a coordinate grid. In the design above, the triangles labeled 1 and 3 are size-change images of triangles labeled 2 and 4. In fact, the entire “ring” of yellow and purple triangles containing 1 and 3, and the entire “ring” of yellow and purple triangles containing 2 and 4 are size-change images of one another. The center
of these size changes is at the center of the design. The lines through the corresponding preimage and image vertices of each triangle are concurrent at the center of the design.

### Activity 2

**MATERIALS** ruler, protractor

![Diagram of triangles with vertices labeled A, B, C, D, E, F, G, and H, with numbers 1, 2, 3, and 4 indicating triangles.

**Step 1** Determine the magnitude of the size-change transformation that takes triangles 1 and 3 to triangles 2 and 4.

**Step 2** Measure the angles in triangles 1, 2, 3, and 4. What do you notice about these angles?

**Step 3** Place a ruler so that one edge of it is a transversal to $\overline{BC}$ and $\overline{FG}$. Use a protractor to measure an angle where the transversal crosses $\overline{BC}$ and the corresponding angle where the transversal crosses $\overline{FG}$. What does this tell you about $\overline{BC}$ and $\overline{FG}$?

**Step 4** Measure the lengths of the sides of triangle 3 and the corresponding sides of triangle 4. What is the ratio of these lengths?

The size-change image of any line segment is a parallel line segment and the size-change image of any angle is an angle with the same measure. You should have seen instances of both of these properties in Activity 2. You should also have seen that the distance between two image points under a size change of magnitude $k \neq 0$ is $k$ times the distance between their preimages. This distance property also applies to the distance between a point of the figure being transformed and the center of the size change—if $|k| > 1$, the image is larger and farther from the center of the size change, and if $|k| < 1$, the image is smaller and closer to the center of the size change.
Example 2

The lengths of the sides of $\triangle QRS$ are 12 cm, 16 cm, and 20 cm. Find the perimeter of the image of $\triangle QRS$ under a size change of magnitude 1.75.

Solution

Under a size change of magnitude 1.75:

the length of the image of the side of $\triangle QRS$ that has length 12 cm is $1.75 \cdot 12 \text{ cm} = \ ? \text{ cm}$;

the length of the image of the side of $\triangle QRS$ that has length 16 cm is $\ ? \cdot 16 \text{ cm} = \ ?$; and

the length of the image of the side of $\triangle QRS$ that has length 20 cm is $\ ? \cdot \ ? = \ ?$.

So, the perimeter of the image of $\triangle QRS$ is $\ ? + \ ? + \ ? = \ ?$.

Note that the perimeter of $\triangle QRS$ is $12 + 16 + 20 = 48 \text{ cm}$ and the perimeter of the image of $\triangle QRS$ is 84 cm, which is $1.75 \cdot 48$.

Questions

Covering the Ideas

In 1–3, refer to Activity 1 in this lesson.

1. a. Fill in the Blanks

   The slope of $\overrightarrow{BC}$ is $\ ?$ and the slope of $\overrightarrow{B'C'}$ is $\ ?$.

   b. What can you conclude about $\overrightarrow{BC}$ and $\overrightarrow{B'C'}$?

2. a. Find $AD$.

   b. Find $A'D'$.

   c. How are $\overline{AD}$ and $\overline{A'D'}$ related?

3. Give an argument that shows that the line through $H$ and $H'$ also contains $(0, 0)$.

4. What is the image of $(18, 33)$ under a size change of magnitude $\frac{2}{3}$?

5. If $(-15, 35)$ is the size-change image of $(-6, 14)$, what is the magnitude of the size change?

6. If $MN = 29 \text{ cm}$, how long is the image of $\overline{MN}$ under a size change of magnitude 4?

7. The perimeter of $\triangle KQX$ is 37. $\triangle K'Q'X'$ is the size-change image of $\triangle KQX$ and has perimeter 11.1. What is the magnitude of the size change?

8. Let $P = (-4, 0)$, $Q = (-2, 6)$, $R = (8, 6)$, and $S = (6, 0)$.

   a. What kind of quadrilateral is $PQRS$?

   b. Find $P'Q'R'S'$, the image of $PQRS$ under a size change of magnitude 0.25.

   c. What kind of quadrilateral is $P'Q'R'S'$? Explain how you know.
APPLYING THE MATHEMATICS

9. Refer again to Activity 1 in this lesson. What is the magnitude of a size change that would map the larger figure back onto the original smaller figure?

10. Let \( U = (12, -4) \) and \( V = (2, 10) \).
   a. Graph \( U \) and \( V \).
   b. Graph \( U' \) and \( V' \), the images of \( U \) and \( V \) under a size change of magnitude 1.5.
   c. Find an equation for \( \overrightarrow{UU'} \).
   d. Find an equation for \( \overrightarrow{VV'} \).
   e. From Parts c and d, determine the point of intersection of \( \overrightarrow{UU'} \) and \( \overrightarrow{VV'} \).

11. Given two points and their images under a size change, describe how to find the center of the size change.

12. Suppose \( ABCD \) is a square and \( A'B'C'D' \) is the size-change image of \( ABCD \) under a size change of magnitude 2.8. Also suppose \( A'B'C'D' \) has perimeter 42.
   a. What is the perimeter of \( ABCD \)?
   b. What is the area of \( A'B'C'D' \)?
   c. What is the area of \( ABCD \)?
   d. What is the ratio of your answer in Part c to your answer in Part b? How does this relate to 2.8?

13. In 2006, the Linux Users Group at Oregon State University created a 220-foot wide “crop circle” of the Mozilla Firefox logo in a field of oats. To do this, they drew a grid like the one shown at the right on top of a copy of the logo, letting the distance between each circle be 2 feet (every 10th circle is drawn thicker to make counting easier). Using this grid, they were able to create the large-scale copy of the logo by creating a size-change image of each section in the field of oats.
   a. While the logo ended up being 220 feet wide, the grid covers a larger area. What is the width of the area represented by the grid?
   b. If the grid they used was 30 inches wide, what was the magnitude of the size change applied to each region?

\[ 0 < k < 1 \]