Lesson 14-9 Similarity

Vocabulary

similar

similarity transformation

BIG IDEA Figures are similar if and only if they are related by a congruence transformation, a size change, or a combination of congruence transformations and size changes.

In Lesson 5-10, we noted that if two figures are similar, then ratios of corresponding lengths are equal and corresponding angles have the same measure (the Fundamental Property of Similar Figures). This idea of similar figures was used to find unknown lengths in one of two similar figures.

When are two figures similar? Congruent figures are figures with the same size and shape; **similar** figures have the same shape, but not necessarily the same size. A **similarity transformation** is a transformation under which the image and preimage are similar. Similarity transformations can be created by combining congruence transformations and size-change transformations.

Activity

MATERIALS ruler, protractor

Step 1 Draw a triangle and label its vertices *A*, *B*, and *C*.

Step 2 Draw a line segment that is not the same length as $\overline{AB}$ and label its endpoints *D* and *E*.

Step 3 Measure $\angle BAC$. Draw a ray $\overrightarrow{DX}$ so that $m\angle EDX = m\angle BAC$. Measure $\angle ABC$. Draw a ray $\overrightarrow{EY}$ so that $\overrightarrow{EY}$ intersects $\overrightarrow{DX}$ and $m\angle DEY = m\angle ABC$.

Step 4 Label the intersection of the rays from Step 3 *F*.

Step 5 Without measuring, how is the measure of angle *ACB* related to the measure of angle *DFE*? How do you know?

Step 6 Measure the sides of $\triangle ABC$ and the sides of $\triangle DEF$. Compute the ratios $\frac{AB}{DE}$, $\frac{AC}{DF}$, and $\frac{BC}{EF}$. What do you notice? How are $\triangle ABC$ and $\triangle DEF$ related?

Because the sum of the measures of the angles in any triangle is 180°, you only need the measures of two angles to determine the measure of the third angle. The Activity suggests a way to tell when two triangles are similar.

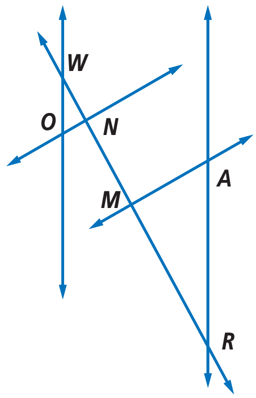
AA Similarity Theorem

If two angles of one triangle have the same measures as two angles of another triangle, then the triangles are similar.

GUIDED

Example 1

$\overleftrightarrow{WO} \parallel \overleftrightarrow{AR}$ and $\overleftrightarrow{NO} \parallel \overleftrightarrow{AM}$. Explain why $\triangle OWN$ is similar to $\triangle ARM$.

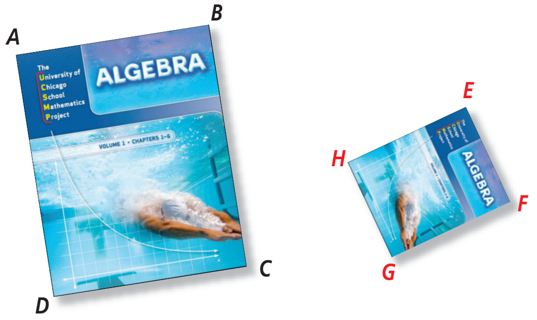


Solution $\overleftrightarrow{WO} \parallel$ ___?___, so m∠OWN = m∠ARM because they are alternate interior angles formed by parallel lines. $\overleftrightarrow{NO} \parallel$ ___?___, so m∠ONW = ___?___ because they are ___?___. Since two angles of △___?___ have the same measure as two angles of △___?___, the triangles are similar by the ___?___.

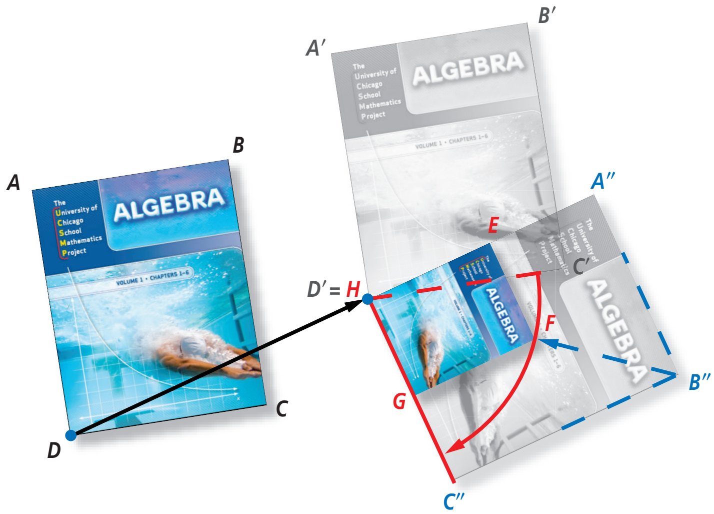
Two figures P and Q are similar, written $P \sim Q$, if and only if they are related by a similarity transformation. So in Example 1 above, $\triangle OWN \sim \triangle ARM$.

Example 2

The two figures below (pictures of your Algebra book) are similar. Describe a similarity transformation that maps *ABCD* onto *EFGH*.



Solution

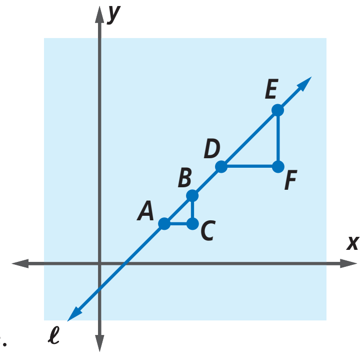


The left figure can be slid so that one corner point matches up with the right figure, then turned to get the sides of that corner to match up, and finally resized to get the whole figure to match. So, using a translation, then a rotation, then a size change maps one figure onto the other.

Here is a more detailed description.

1. Translate $ABCD$ so that $D' = H$. The image is $A'B'C'D'$.
2. Rotate $A'B'C'D'$ about D' so that the image $A''B''C''D'$ is such that the rays $\overrightarrow{D'E}$ and $\overrightarrow{D'A''}$ coincide.
3. Measure the lengths of corresponding sides $\overline{AB}$ and $\overline{EF}$. Find the ratio $\frac{EF}{AB}$. We find that $\frac{EF}{AB} = 0.5$.
4. Apply a size change of magnitude 0.5 to $A''B''C''D'$.

Similarity and the Slope of a Line



You have learned that the slope of a line through (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$. For a given line, does it mattter which two points you use to determine the slope of the line? Here is a proof that any pair of points will yield the same slope.

Given line ℓ, choose any four points A, B, D, and E. Draw $\triangle ABC$ and $\triangle DEF$ so $\overline{AC}$ and $\overline{DF}$ are parallel to the x-axis and $\overline{EF}$ and $\overline{BC}$ are parallel to the y-axis. This also means that $\overline{AC} \parallel \overline{DF}$ and $\overline{BC} \parallel \overline{EF}$.

$m\angle BAC = m\angle EDF$	If parallel lines are cut by a transversal, then corresponding angles have the same measure.
$m\angle ABC = m\angle DEF$	
$\triangle ABC \sim \triangle DEF$	AA Similarity Theorem
$\frac{BC}{EF} = \frac{AC}{DF}$	Fundamental Property of Similar Figures
$BC \cdot DF = AC \cdot EF$	Means-Extremes Property
$\frac{BC}{AC} = \frac{EF}{DF}$	Divide both sides by $AC \cdot DF$.

So the slope calculated with any pair of points on a given line is constant.

Questions

COVERING THE IDEAS

In 1 and 2, True or False.

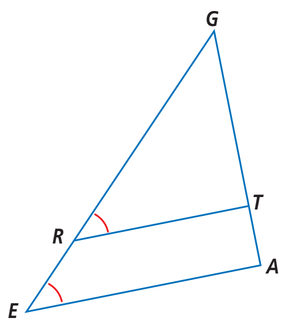
1. All congruent figures are similar.
2. All similar figures are congruent.

In 3 and 4, a figure is given.

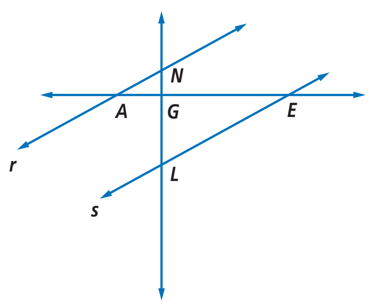
a. Write a triangle similarity sentence.

b. List the angles with equal measures that support the similarity statement.

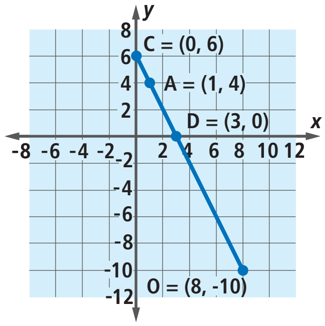
3.



4. $r \parallel s$

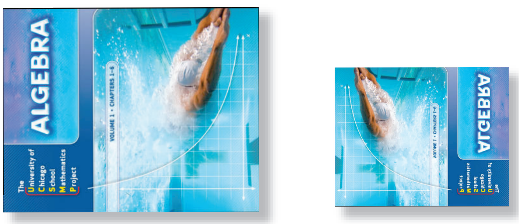


5. Copy the diagram at the right.
   a. Plot T so $\triangle CAT$ is a right triangle.
   b. Plot G so $\triangle DOG$ is a right triangle.
   c. Calculate $\frac{CT}{TA}$.
   d. Calculate $\frac{DG}{GO}$.
   e. Show $\frac{CT}{TA} = \frac{DG}{GO}$.
   f. What does Part e show?



In 6 and 7, describe transformations whose combination is a similarity transformation that maps one image onto the other.

6.



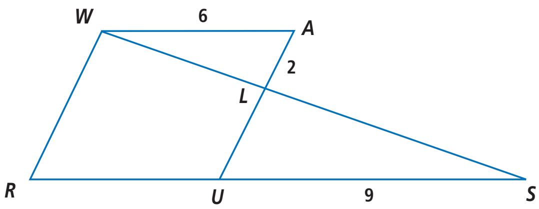
7.

APPLYING THE MATHEMATICS

8. The points (0, -3), (10, 5), (-5, -7), and (1, -2.2) all lie on the same line ℓ.
   a. Use 2 pairs of points to determine the slope of ℓ.
   b. Show both slopes are equal.
   c. What is the equation of the line?

9. Explain why all equilateral triangles are similar.

10. *WAUR* is a parallelogram.



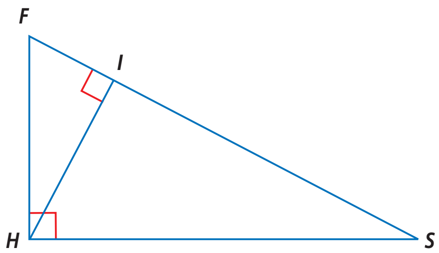
a. Name all pairs of similar triangles.

b. Find *LU*.

c. Find *RS*.

d. Find *WR*.

11. $\triangle FSH$ is a right triangle with $\overline{FH} \perp \overline{HS}$ and $\overline{HI} \perp \overline{FS}$.



a. Why is $\triangle HFI$ similar to $\triangle SFH$?

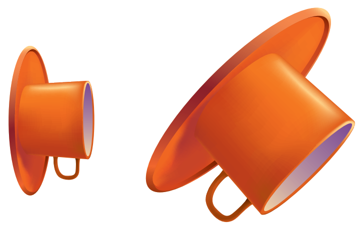
b. Why is $\triangle HIS$ similar to $\triangle FHS$?

c. Name a third pair of similar triangles.

In 12 and 13, describe transformations whose combination is a similarity transformation that maps one image onto the other.

12.

13.



In 14 and 15, use the following theorem:

Three points are collinear if and only if the slopes between each pair of points are equal.

14. Which of the points $B = (0, -4)$, $I = (1, -3)$, $R = (6, 0)$, $D = (-3, -6)$ is not collinear with the other three?

15. If $(0, 3)$, $(2, -1)$, $(x, 1)$, and $(5, y)$ are collinear, find x and y.