The Golden Gate Bridge is a suspension bridge across the opening of San Francisco Bay as it joins the Pacific Ocean. It is one of the most beautiful, and the most photographed, bridges in the world. The highest point of the bridge is the top of a tower, 227 meters above the water. The foundation of the south tower is 34 meters below the mean water level.

Example 1
Find the height of the bridge from its highest point to its lowest point.

Solution
Step 1 Represent the highest and lowest points of the bridge on a vertical number line.

Step 2 Subtract the coordinates, the greater minus the lesser.
   
   \[227 - (-34) = ?\]

The total height of the Golden Gate Bridge is about \(?\).
Lesson 5-7A

Generalizing the Example

In Example 1, two points were graphed on a number line and the distance between them was found by subtracting the lesser from the greater. In general, to find the distance between two points, \( a \) and \( b \), the answer can be obtained by subtracting either number from the other. For instance, in Example 1, we could subtract \( 227 - (-34) = 261 \) or \( -34 - 227 = -261 \). However, distances are represented only by positive numbers or zero. Therefore, if the difference is positive or zero, you have your answer. If the difference is negative, then you want its opposite. Recall from Lesson 1-8A that this is exactly what absolute value does.

Distance on a Number Line

The distance between points \( a \) and \( b \) is \(| a - b |\).

STOP QY

Notice that if you change the order of subtraction before taking the absolute value, you will not change the distance between the points. In general, for any numbers \( a \) and \( b \),

\[ |a - b| = |b - a|. \]

Horizontal and Vertical Distances on a Coordinate Grid

On a coordinate grid, you can determine horizontal and vertical distances by counting spaces or by subtracting appropriate coordinates and taking the absolute value.

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Example 2

Find the lengths of \( \overline{AB} \) and \( \overline{BC} \) in \( \triangle ABC \) at right.

Solution 1 Since \( \overline{AB} \) is a horizontal line segment, start at \( A \) and count the spaces horizontally to \( B \). \( \overline{AB} \) is \( ? \) units long.

Since \( \overline{BC} \) is a vertical line segment, start at \( B \) and count the spaces vertically to \( C \). \( \overline{BC} \) is \( ? \) units long.

Solution 2 \( A \) and \( B \) have different \( x \)-coordinates but the same \( y \)-coordinate, which means \( \overline{AB} \) is a \( ? \) line segment. To find its length, subtract the \( x \)-coordinates and take the absolute value.

\[ |4 - (-3)| = \_\_ \text{ or } |-3 - \_\_| = \_. \]

\( B \) and \( C \) have different \( y \)-coordinates but the same \( x \)-coordinate, which means \( \overline{BC} \) is a \( ? \) line segment. To find its length, subtract the \( y \)-coordinates and take the absolute value.

\[ |-2 - 5| = \_\_ \text{ or } \_\_ - (-2) = \_. \]
Example 3

Find the distance between \(U\) and \(P\) if \(U = (-3, 2)\) and \(P = (4, 2)\).

Solution 1: Draw a picture. Since \(UP\) is a horizontal line segment, start at \(U\) and count the spaces along the x-axis to \(P\). \(UP\) is 7 units long.

Solution 2: \(U\) and \(P\) have different x-coordinates but the same y-coordinate, which means \(UP\) is horizontal. To find its length, subtract the x-coordinates and take the absolute value.

\[|\text{-3} - \text{4}| = |\text{-7}| = \text{7}\] or \[|\text{4} - (\text{-3})| = |\text{7}| = \text{7}\].

Questions

COVERING THE IDEAS

1. Jefferson Middle School has a basement that reaches 18 feet underground. The two-story building rises 30 feet above ground.
   a. Represent the highest and lowest points of the building on a vertical number line.
   b. Write an expression using absolute value for the total height of the building.
   c. Find the total height of Jefferson Middle School.

In 2–4, two points on a number line are given.
   a. Write an expression using absolute value for the distance from one point to the other.
   b. Find the distance between the points.

2. 

3. 

4. 

5. Points \(P\) and \(Q\) are graphed on a number line with coordinates \(m\) and \(n\). What is the distance between \(P\) and \(Q\)?

6. Given \(G = (6, 8)\) and \(H = (6, -3)\):
   a. Graph \(G\) and \(H\).
   b. Find the length of \(GH\) two ways.
In 7–9, coordinates of two points are given.
   a. Tell whether the line containing the points is horizontal or vertical.
   b. Find the distance between the points.
   7. (5, -2), (23, -2)
   8. (8.2, 4.3), (-3.7, 4.3)
   9. (\(3\frac{1}{2}, 2\frac{1}{4}\)), (\(3\frac{1}{2}, -1\frac{1}{2}\))

**APPLYING THE MATHEMATICS**

10. For a football play, consider the line of scrimmage (the starting position of the ball) as 0. The quarterback dropped back 11 yards to throw a pass. The receiver caught the ball 16 yards beyond the line of scrimmage and was tackled immediately.
    a. Draw a horizontal number line showing the position of the quarterback and receiver as the pass is being executed.
    b. Write an expression for how far the pass traveled in the air.
    c. How long was the pass?

11. a. Find \(AB\) in two ways.

   ![Diagram of A and B points]

   b. What adjustment is necessary because each space is 3 units long?

In 12 and 13, use the following information. The streets of Chicago are organized like a coordinate grid, with the origin at the corner of State Street, which runs north and south, and Madison Street, which runs east and west. Addresses one mile apart have numbers differing by 800.
    a. Draw a coordinate grid for plotting the given landmarks.
    b. Write a problem for determining the distance between them.
    c. How far is it from one landmark to the other?

12. The Art Institute: (125, -100) and the Chicago Water Tower: (125, 800)

13. Chicago City Hall: (-300, 200) and the Chicago Cultural Center: (100, 200)

In 14 and 15, find the perimeter of a rectangle with the given vertices.

14. (-2, 4), (8, 4), (8, -2), (-2, -2)

15. (7.5, 12.6), (-2.3, 12.6), (-2.3, 18.4), (7.5, 18.4)
16. Below is a table showing the highest and lowest elevations in Europe and North America.

<table>
<thead>
<tr>
<th></th>
<th>Highest</th>
<th>Lowest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Europe</td>
<td>Mt. Elbrus, Russia</td>
<td>Caspian Sea, Russia</td>
</tr>
<tr>
<td></td>
<td>18,510 ft</td>
<td>–92 ft</td>
</tr>
<tr>
<td>North America</td>
<td>Mt. McKinley, Alaska</td>
<td>Death Valley, California</td>
</tr>
<tr>
<td></td>
<td>20,320 ft</td>
<td>–282 ft</td>
</tr>
</tbody>
</table>

Source: http://www.infoplease.com/lpa/A0001763.html

a. Find the distance from the highest to lowest elevation in both North America and Europe.
b. Which continent has the greater variation? How much greater?

17. \( W \) and \( Z \) are two points on a number line. \( W = -5 \).
   If \( |W - Z| = 11 \), what are the possible coordinates of \( Z \)?