BIG IDEA
The number of times an event will most likely occur is the product of the probability of the event and the number of possible times it could occur.

What is a Probability Model?
A probability model is a list of all possible outcomes in a situation and a probability for each outcome. For example, a probability model for the outcomes when tossing a fair coin is shown in the table below.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>heads</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>tails</td>
<td>( \frac{1}{2} )</td>
</tr>
</tbody>
</table>

In developing a probability model, first list all of the possible outcomes. Next, assign a probability to each outcome.

QY1

In Lesson 3-9, you saw that there are three common ways to determine probabilities of outcomes. The probability model for the coin is based on assuming that heads and tails are equally likely.

Another way of developing a probability model is to use actual data about the situation. For instance, your blood type determines whether your blood can be given to someone else or received from someone else in a blood transfusion. People in the world have blood type A, B, AB, or O. In the United States as a whole, about 38.8% have type A, 11.1% have type B, 3.9% have type AB, and 46.1% have type O. This means that if a person is randomly selected in the United States, there is a 38.8% probability that the person’s blood is of type A.

QY2

What is the probability that a person in the U.S. has blood of type O?
Example 1
Create a probability model for the blood type situation in the U.S.

Solution  
Take the percents as the probabilities of a person being of each blood type.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.388</td>
</tr>
<tr>
<td>B</td>
<td>0.111</td>
</tr>
<tr>
<td>AB</td>
<td>0.039</td>
</tr>
<tr>
<td>O</td>
<td>0.461</td>
</tr>
</tbody>
</table>

Example 2
Use the probability model from Example 1. A hospital wants to have enough blood to handle 60 transfusions at a given time. If 60 different people were to receive these transfusions, about how many would you expect to be of each blood type?

Solution  
Remember that the probabilities in Example 1 are percents of people in the U.S. who have each blood type. If the probability that a person is of type A is 0.388, then of 60 people we would expect 0.388 \times 60 = 23.28, or about 23 to be of type A. In a group of 60 people, we would expect \( \? \times 60 \), or about \( \? \), to be of type B. We would expect \( \? \) to be of type AB and \( \? \) to be of type O.

Check  
Add the expected numbers of each type. 23 + \( \? \) + \( \? \) + \( \? \). You should get 60.

The numbers in the answer to Guided Example 2 are expected counts. Think of the blood type of each person in Example 1 and 2 as being the outcome of an event in a blood-type experiment. The 60 people become 60 repeated trials of the experiment. If an event in an experiment has probability \( p \), then in \( n \) trials the expected count of the event is determined by finding the product \( p \times n \). So, the expected count of people with type A blood in a group of 60 randomly selected people in the U.S. is 23.28.

An expected count does not tell you exactly what will happen. It gives you the single most likely outcome, but there are many other possibilities. While 23 is the best prediction in the situation above, it would not be surprising to have 24, 27, 19, or some other number of people with type A blood. It would not be impossible to have only 3 people with type A blood, but it would be very unlikely.
Example 3
A jar contains 3 red marbles, 5 blue marbles, and 2 green marbles. You draw one marble randomly from the jar.

a. Develop a probability model for the color of the marble that is selected.
b. Use your model to give the expected count of red marbles if you repeated this experiment 100 times, replacing the marble each time.

Solution

a. First make a list of all possible outcomes. The possible outcomes for the color of the marble drawn are red, __, and __.
Next, assign a probability to each outcome. Assuming it is equally likely that you draw any given marble, you can use the Probability Formula.
There are 3 red marbles out of a total of __ marbles, so \( P(\text{red}) = \frac{3}{7} \).
There are __ blue marbles out of a total of __ marbles, so \( P(\text{blue}) = \frac{2}{7} = \frac{2}{7} \).
There are __ green marbles out of a total of __ marbles, so \( P(\text{green}) = \frac{2}{7} = \frac{2}{7} \).

b. The probability of picking a red marble is \( \frac{3}{7} \) and if you were to choose a marble 100 times, the expected count would be \( \frac{3}{7} \cdot 100 = \frac{300}{7} \).

Activity

Work with a partner with a newspaper, magazine, or trade book. In this activity, you will make predictions about the frequency of letters in everyday written English.

Step 1 Select several paragraphs of text. Record in a table the frequency of each letter in the selected section of text. Do you think the outcomes (the occurrence of letters) are equally likely?

Step 2 Which three letters occurred with the greatest frequency?

Step 3 Find the relative frequency for each of the three letters in Step 2. Do not use spaces, punctuation marks, or numerals in your calculations.

Step 4 Use your relative frequencies to calculate the expected count for each of those three letters in a 13,250-word story.
Questions

COVERING THE IDEAS

1. In the situation of Examples 1 and 2, if 17 people are injured in a train wreck seriously enough to need blood, about how many would be expected to be each blood type?

2. Consider the jar of marbles from Example 3. Give the expected count of each event.
   a. Choosing a red marble in 200 trials.
   b. Choosing a blue marble in 10,000 trials.
   c. Choosing a green marble in 1,000,000 trials.

3. **Multiple Choice** Suppose the five regions of the spinner at the right are equally likely. If the spinner is spun 800 times, what is true about the expected count for obtaining a prime number?
   A  A prime number will occur exactly 480 times.
   B  You are almost certain to get a prime number exactly 480 times.
   C  A prime number is likely to occur about 480 times.
   D  If you do not obtain a prime number exactly 480 times, there is something wrong with the spinner.

4. Suppose about 70 days each year are sunny in Portland, Oregon. If you were to visit Portland for a 30-day month, about how many days should you expect to be sunny?

5. Giacomo is taking a multiple-choice test that has four possible answer choices for each item, A, B, C, and D. When time runs out there are 20 questions left on the test, so he simply guesses at all 20 answers.
   a. Predict about how many questions Giacomo can expect to answer correctly.
   b. Suppose the test had five possible answer choices for each item, A, B, C, D, and E. Give the expected number of correct answers in the 20 questions.

6. Fifty students were asked in a poll to name their preferred candidate for class president in an upcoming election. The results are shown at the right.
   a. Does it appear that the candidates are likely to each receive the same number of votes in the actual election?
   b. Find the relative frequency for each candidate.
   c. Use the relative frequencies to predict about how many votes each candidate is likely to get if 400 students vote in the actual election.
7. An insurance company has a goal for each agent to sell at least 12 policies this month. Suppose an agent has a relative frequency of selling 1 policy in 35 customer calls. Is the agent likely to meet the goal if he or she makes 500 calls this month? Explain your answer.

8. In Question 3, suppose you spun the spinner to find the relative frequency of the number of 2s in 1000 spins. Would you expect your relative frequency to be closer to \(\frac{1}{5}\) than a relative frequency obtained from 100 spins? Why or why not?

9. At last year’s school picnic, 105 hot dogs, 68 hamburgers, and 85 slices of pizza were served to 150 students. This year, 200 students are expected to attend the picnic. About how much of each type of food should the school expect to serve if the eating habits of students remain the same as last year?

10. You will need a paper cup to complete this question.
   a. If you toss a paper cup in the air, it may land in any of these three positions. Do you think the outcomes are equally likely? Explain.
   
   
   **Bottom**
   **Top**
   **Side**

   b. Toss the cup 25 times, and record each result. Then write the relative frequency for each possible outcome.

   c. Use your relative frequencies to predict about how many times each outcome is likely to occur in 500 tosses of the cup.

   **QY ANSWERS**

   1. | Outcome | Probability |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\frac{1}{6})</td>
</tr>
<tr>
<td>2</td>
<td>(\frac{1}{6})</td>
</tr>
<tr>
<td>3</td>
<td>(\frac{1}{6})</td>
</tr>
<tr>
<td>4</td>
<td>(\frac{1}{6})</td>
</tr>
<tr>
<td>5</td>
<td>(\frac{1}{6})</td>
</tr>
<tr>
<td>6</td>
<td>(\frac{1}{6})</td>
</tr>
</tbody>
</table>

   2. 46.1%